



#### Hydrostatic Equilibrium

First order for a spherical body: Internal structure is determined by the balance between gravity and pressure:

$$P(r) = \int_{r}^{R} g_{P}(r')\rho(r')dr'$$

Which is solvable if the density profile is known. If we assume that the density is constant throughout the planet, we obtain a simplified relationship for the central pressure:  $3GM^2$ 

$$P_C = \frac{3GM^2}{8\pi R^4}$$

\* Which is really just a lower limit since we know generally density is higher at smaller radii.

#### Equation of State

To predict the density of a pressurized material, need *Equation of State* relating density, temperature and pressure of a material.

At low pressure, non-interacting particles  $\rightarrow$  "ideal gas"

For the high temperature/pressure interiors of planets, it's often written as a power law:

 $P = K \rho^{(1+1/n)}$ 

Where n is the polytropic index: ∞ for isothermal sphere, ~3/2 for planets, ~3 for main-sequence stars.

These power laws are obtained empirically from measurements in shock waves, diamond anvil cells.





#### **Planetary Thermal Profiles**

- A planet's thermal profile is driven by the sources of heat:
- Accretion
- Differentiation
- **Radioactive decay**
- **Tidal dissipation**
- as well as the modes and rates of heat transport:
- Conduction
- Radiation
- **Convection/advection**

### Europa – conduction or convection?

(Schmidt et al., 2011)





#### **Material Properties**

Knowing the phases and physical/chemical behavior of materials that make up planetary interiors is also extremely important for interior models:

However, these properties are not well known at the extreme temperatures and pressures found at depth, so we rely on experimental data that can empirically extend our understanding of material properties into these regimes:





#### **Material Properties**

For gas giants like Jupiter and Saturn, understanding how hydrogen and helium behave and interact is critically important for modeling their interiors.

At > 1.4 Mbar, molecular fluid hydrogen behaves like a metal in terms of conductivity, and it's believed that this region is where the convective motion drives the magnetic dynamo



#### **Material Properties**

For ice giants like Uranus and Neptune, understanding how water and ammonia behave and interact is critically important for modeling their interiors.

At high pressures, the liquid state of water becomes a supercritical fluid (i.e. not a true gas nor liquid) with higher conductivity







#### **Gravity Fields**

**Gravitational Potential:** 

$$\Phi_{g}(r,\phi,\theta) = -\left(\frac{GM}{r} + \Delta\Phi_{g}(r,\phi,\theta)\right)$$

Where the first term represents a non-rotating fluid body in hydrostatic equilibrium, and the second term represents deviations from that idealized scenario.

#### In the most general case:

$$\Delta \Phi_g(r,\phi,\theta) = \frac{GM}{r} \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{R}{r}\right)^n \left(C_{nm} \cos m\phi + S_{nm} \sin m\phi\right) P_{nm}(\cos\theta)$$

Where  $C_{nm}$  and  $S_{nm}$  are determined by internal mass distribution, and  $P_{nm}$  are the Legendre polynomials

$$\Delta \Phi_g(r,\phi,\theta) = \frac{GM}{r} \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{R}{r}\right)^n \left(C_{nm} \cos m\phi + S_{nm} \sin m\phi\right) P_{nm}(\cos\theta)$$

This can be greatly simplified for giant planets with a few assumptions:

Axisymmetric about the rotation axis  $\rightarrow S_{nm}$ ,  $C_{nm}$ = 0 for m > 0 (reasonable since most departures from a sphere are due to the centrifugally driven equatorial bulge)

$$\Phi_{g}(r,\phi,\theta) = -\frac{Gm}{r} \left[ 1 - \sum_{n=2}^{\infty} J_{n} P_{n}(\cos\theta) \left(\frac{R}{r}\right)^{n} \right].$$
(2.29)

For N-S symmetry (not Mars!), also  $J_n=0$  for odd n

Earth's Interior Structure Hydrostatic Equilibrium Heating Constituent Relations Gravitational Fields

**Isostasy** Magnetism



#### Isostasy



Now apply this idea to topography and the crust...

#### **Airy Model**

#### topography underlain by thick root



High topography (relative to surroundings) due to THICK CRUST Example - Himalayas/Tibet

Courtesy of U of Leeds

# The Earth's Crust

Credit: Wikipedia



 Airy Scheme: Accommodate topography with crustal 'root' (assumes same ρ for all of the crust)

2. Pratt Scheme: Lateral density variation causes topography

The Earth's granitic (lower density) continental crust varies from < 20 km under active margins to ~80 km thick under the Himalayas.

The basaltic (higher density) oceanic crust has an average thickness of 6 km with less near the spreading ridges.





#### Magnetosphere (Chapter 7)

#### Earth's Magnetic Field



Solar Wind



Similar to a "bar magnet", the Earth' s intrinsic field is roughly dipolar.

The solar wind deforms the magnetic field, and creates both a magnetopause and bow shock.



#### The Earth's Magnetic Field Changes in the Earth's magnetic field

#### Drift of the magnetic pole



# Reversal of the field direction recorded in the sea floor



#### The Earth's Magnetic Field



The highly conductive liquid outer core has the capacity to carry the electric currents needed to support a geodynamo Changes observed in the paleomagnetic record of the Earth's magnetic field indicate it can not be a 'permanent magnet'



#### **Other Planetary Magnetic Fields**

Mercury, Earth, Jupiter, Saturn, Uranus and Neptune all have confirmed global magnetic fields sourced internally. To date, Ganymede is the only moon with a dynamo driven magnetic field.







# Mars Remnant Magnetism



Mars currently has no global magnetic field, but clues to a past dynamo are locked in the remnant magnetization of the crust.

- Is this lack of global field responsible for the atmospheric loss?
- Can we date the dynamo 'turn-off' point?

### **Next: Solar System/Planet Formation**

#### Read chapter 13!!



