Moment of Inertia

\[ \frac{L}{\omega_{rot}} = I = 0.4MR^2 \] for a uniform density sphere, but most planets are centrally concentrated, so \( I < 0.4MR^2 \)

For planets at hydrostatic equilibrium, can infer \( I \) from mass, radius, rotation rate and oblateness

<table>
<thead>
<tr>
<th>Body</th>
<th>( \frac{I}{mr^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moon</td>
<td>0.391</td>
</tr>
<tr>
<td>Mars</td>
<td>0.365</td>
</tr>
<tr>
<td>Earth</td>
<td>0.3307</td>
</tr>
<tr>
<td>Neptune</td>
<td>0.29</td>
</tr>
<tr>
<td>Jupiter</td>
<td>0.26</td>
</tr>
<tr>
<td>Uranus</td>
<td>0.23</td>
</tr>
<tr>
<td>Saturn</td>
<td>0.20</td>
</tr>
<tr>
<td>Sun</td>
<td>0.06</td>
</tr>
</tbody>
</table>
Equation of State

To predict the density of a pressurized material, need *Equation of State* relating density, temperature and pressure of a material.

At low pressure, non-interacting particles $\rightarrow$ “ideal gas”

For the high temperature/pressure interiors of planets, it’s often written as a power law:

$$ P = K \rho^{(1+1/n)} $$

Where $n$ is the polytropic index: $\infty$ for isothermal sphere, $\sim 3/2$ for planets, $\sim 3$ for main-sequence stars.

These power laws are obtained empirically from measurements in shock waves, diamond anvil cells.
Planetary Interiors

Earth’s Interior Structure
Hydrostatic Equilibrium
Constituent Relations
Gravitational Fields
Isostasy
Magnetism
Planetary Thermal Profiles

A planet’s thermal profile is driven by the sources of heat:

- Accretion
- Differentiation
- Radioactive decay
- Tidal dissipation

as well as the modes and rates of heat transport:

- Conduction
- Radiation
- Convection/advection
Europa – conduction or convection?

(Schmidt et al., 2011)
Planetary Interiors

Earth’s Interior Structure
Hydrostatic Equilibrium
Heating
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Isostasy
Magnetism
Material Properties

Knowing the phases and physical/chemical behavior of materials that make up planetary interiors is also extremely important for interior models:

However, these properties are not well known at the extreme temperatures and pressures found at depth, so we rely on experimental data that can empirically extend our understanding of material properties into these regimes:

Courtesy: Synchrotron Light Laboratory
For gas giants like Jupiter and Saturn, understanding how hydrogen and helium behave and interact is critically important for modeling their interiors.

At > 1.4 Mbar, molecular fluid hydrogen behaves like a metal in terms of conductivity, and it’s believed that this region is where the convective motion drives the magnetic dynamo.
Material Properties

For ice giants like Uranus and Neptune, understanding how water and ammonia behave and interact is critically important for modeling their interiors.

At high pressures, the liquid state of water becomes a supercritical fluid (i.e. not a true gas nor liquid) with higher conductivity.

Phase Diagram for Water, Modified from UCL
Planetary Interiors

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Isostasy
Magnetism
Gravity Fields

Gravitational Potential:

\[ \Phi_g(r, \phi, \theta) = -\left( \frac{GM}{r} + \Delta \Phi_g(r, \phi, \theta) \right) \]

Where the first term represents a non-rotating fluid body in hydrostatic equilibrium, and the second term represents deviations from that idealized scenario.

In the most general case:

\[ \Delta \Phi_g(r, \phi, \theta) = \frac{GM}{r} \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left( \frac{R}{r} \right)^n \left( C_{nm} \cos m\phi + S_{nm} \sin m\phi \right) P_{nm} \left( \cos \theta \right) \]

Where \( C_{nm} \) and \( S_{nm} \) are determined by internal mass distribution, and \( P_{nm} \) are the Legendre polynomials.
Gravity Fields

\[ \Delta \Phi_g (r, \phi, \theta) = \frac{GM}{r} \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left( \frac{R}{r} \right)^n \left( C_{nm} \cos m\phi + S_{nm} \sin m\phi \right) P_{nm}(\cos \theta) \]

This can be greatly simplified for giant planets with a few assumptions:

Axisymmetric about the rotation axis \( \rightarrow S_{nm}, C_{nm} = 0 \) for \( m > 0 \) (reasonable since most departures from a sphere are due to the centrifugally driven equatorial bulge)

\[ \Phi_g (r, \phi, \theta) = -\frac{Gm}{r} \left[ 1 - \sum_{n=2}^{\infty} J_n P_n(\cos \theta) \left( \frac{R}{r} \right)^n \right]. \]

(2.29)

For N-S symmetry (not Mars!), also \( J_n = 0 \) for odd \( n \)