Moment of Inertia

$L/\omega_{rot} = I = 0.4MR^2$ for a uniform density sphere, but most planets are centrally concentrated, so $I < 0.4MR^2$

For planets at hydrostatic equilbrium, can infer *I* from mass, radius, rotation rate and oblateness

Body	I/mr^2
Moon	0.391
Mars	0.365
Earth	0.3307
Neptune	0.29
Jupiter	0.26
Uranus	0.23
Saturn	0.20
Sun	0.06

Equation of State

To predict the density of a pressurized material, need *Equation of State* relating density, temperature and pressure of a material.

At low pressure, non-interacting particles → "ideal gas"

For the high temperature/pressure interiors of planets, it's often written as a power law:

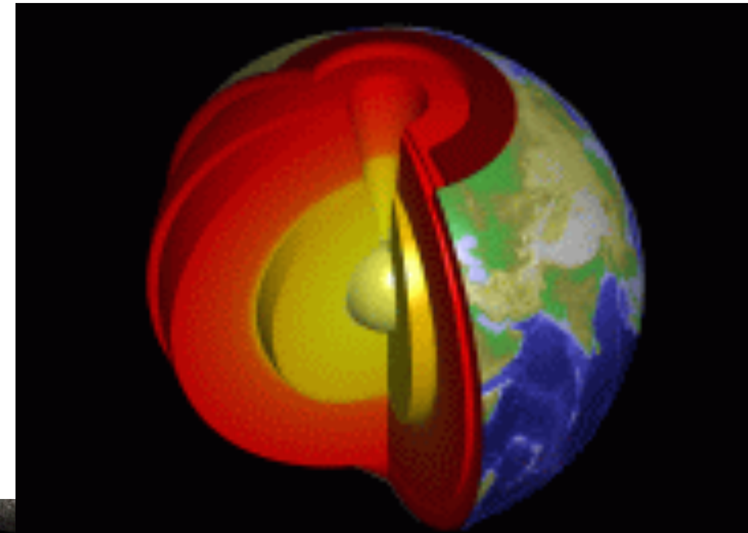
$$P = K\rho^{(1+1/n)}$$

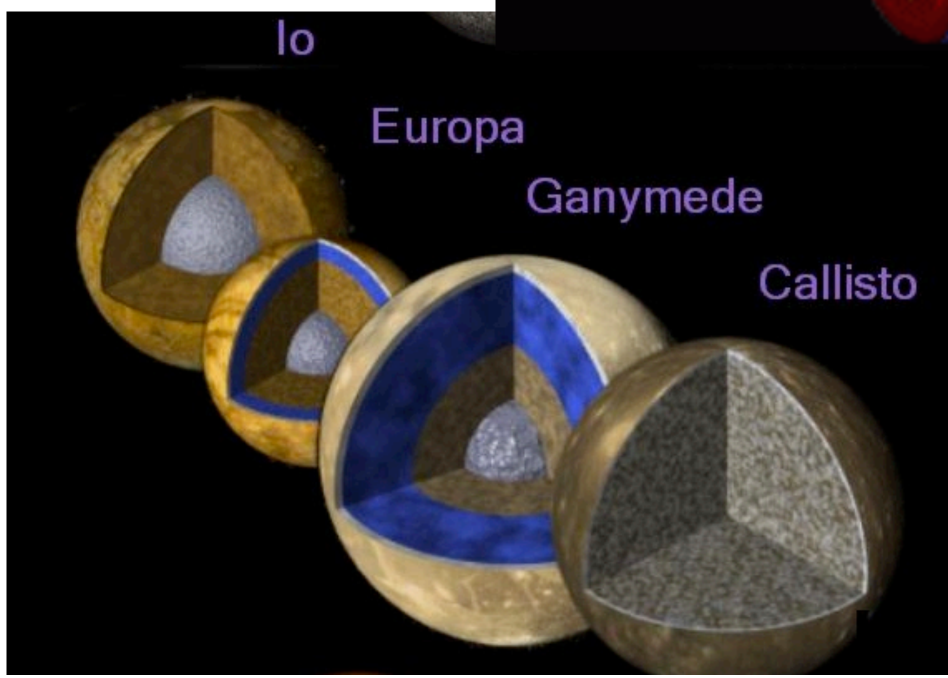
Where n is the polytropic index: ∞ for isothermal sphere, ~3/2 for planets, ~3 for main-sequence stars.

These power laws are obtained empirically from measurements in shock waves, diamond anvil cells.

Planetary Interiors

Earth's Interior Structure
Hydrostatic Equilibrium
Heating
Constituent Relations
Gravitational Fields
Isostasy
Magnetism





Planetary Thermal Profiles

A planet's thermal profile is driven by the sources of heat:

Accretion

Differentiation

Radioactive decay

Tidal dissipation

as well as the modes and rates of heat transport:

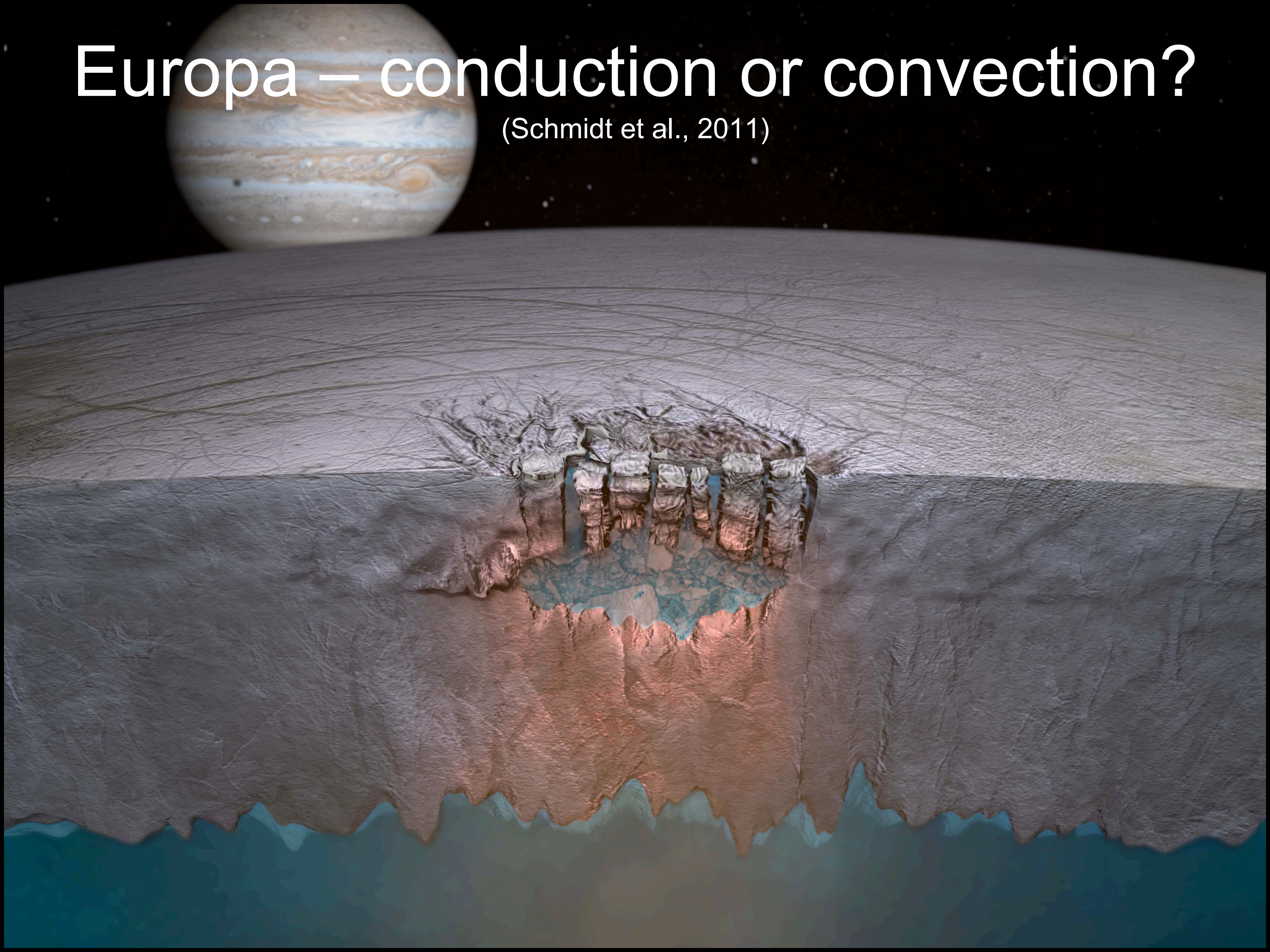
Conduction

Radiation

Convection/advection

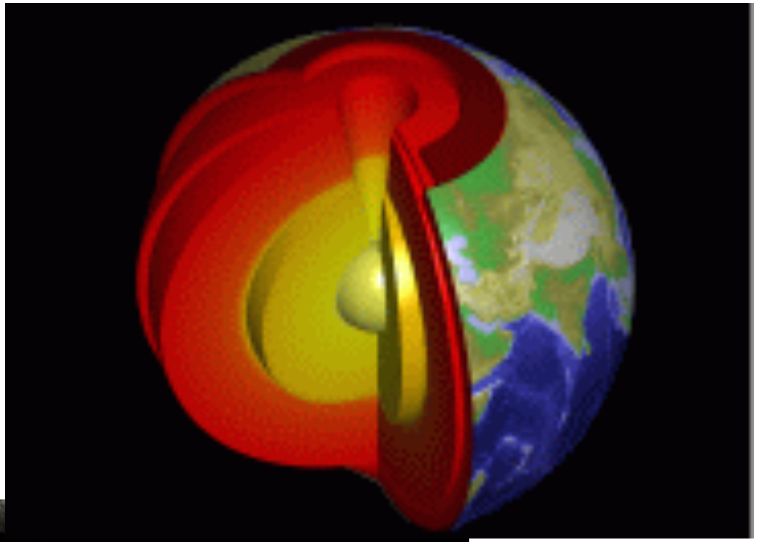
Europa – conduction or convection?

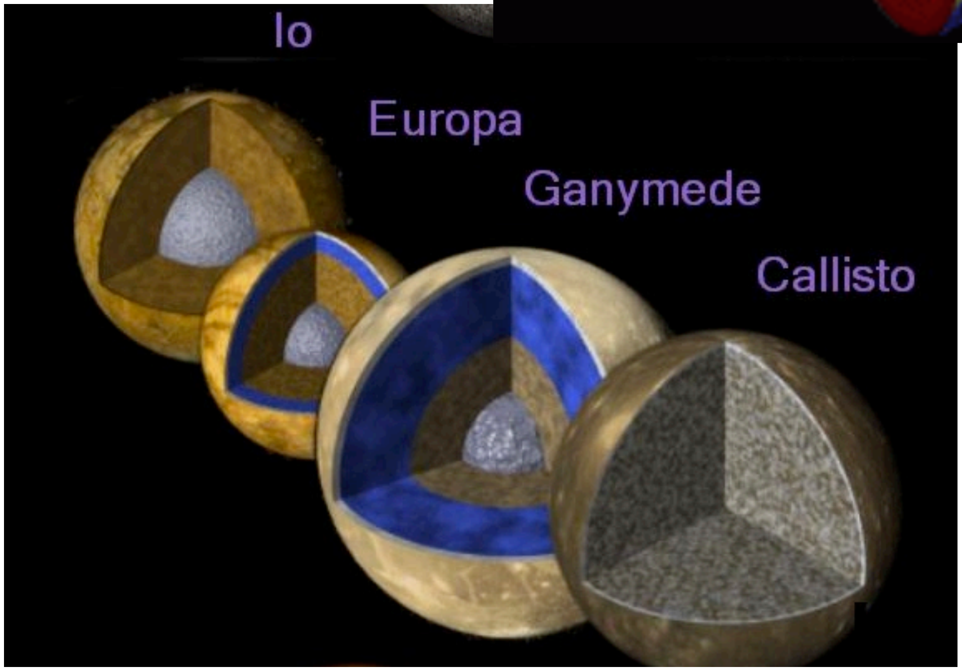
(Schmidt et al., 2011)



Planetary Interiors

Earth's Interior Structure
Hydrostatic Equilibrium
Heating
Constituent Relations
Gravitational Fields
Isostasy
Magnetism

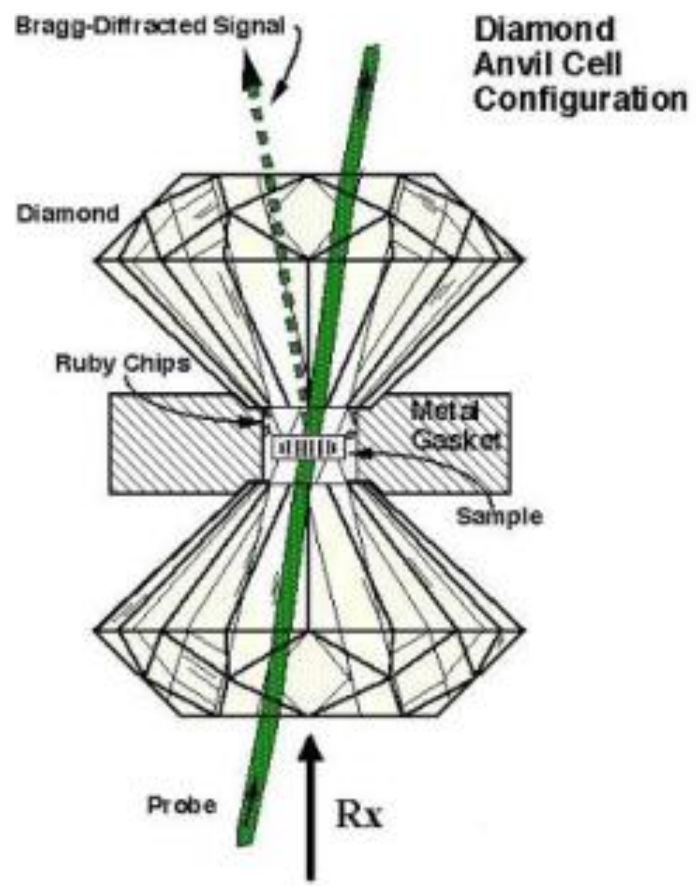




Material Properties

Knowing the phases and physical/chemical behavior of materials that make up planetary interiors is also extremely important for interior models:

However, these properties are not well known at the extreme temperatures and pressures found at depth, so we rely on experimental data that can empirically extend our understanding of material properties into these regimes:

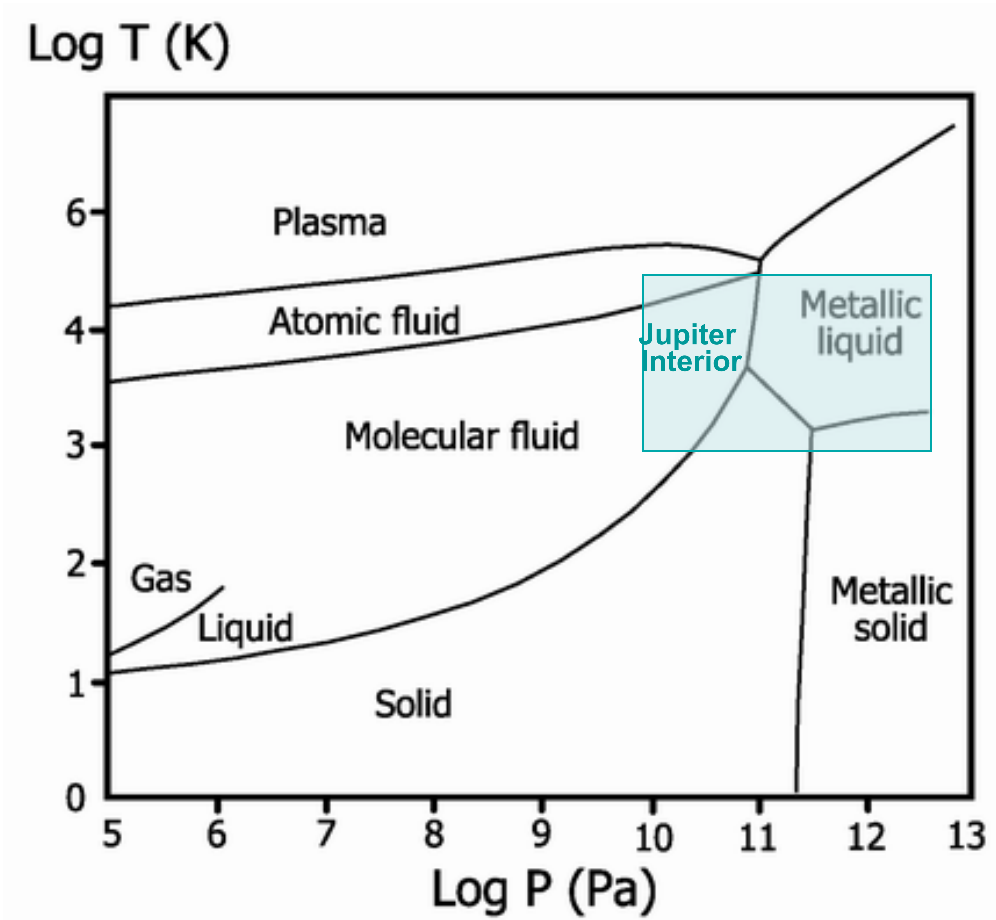


Courtesy:
Synchrotron Light Laboratory

Material Properties

For gas giants like Jupiter and Saturn, understanding how hydrogen and helium behave and interact is critically important for modeling their interiors.

At > 1.4 Mbar, molecular fluid hydrogen behaves like a metal in terms of conductivity, and it's believed that this region is where the convective motion drives the magnetic dynamo

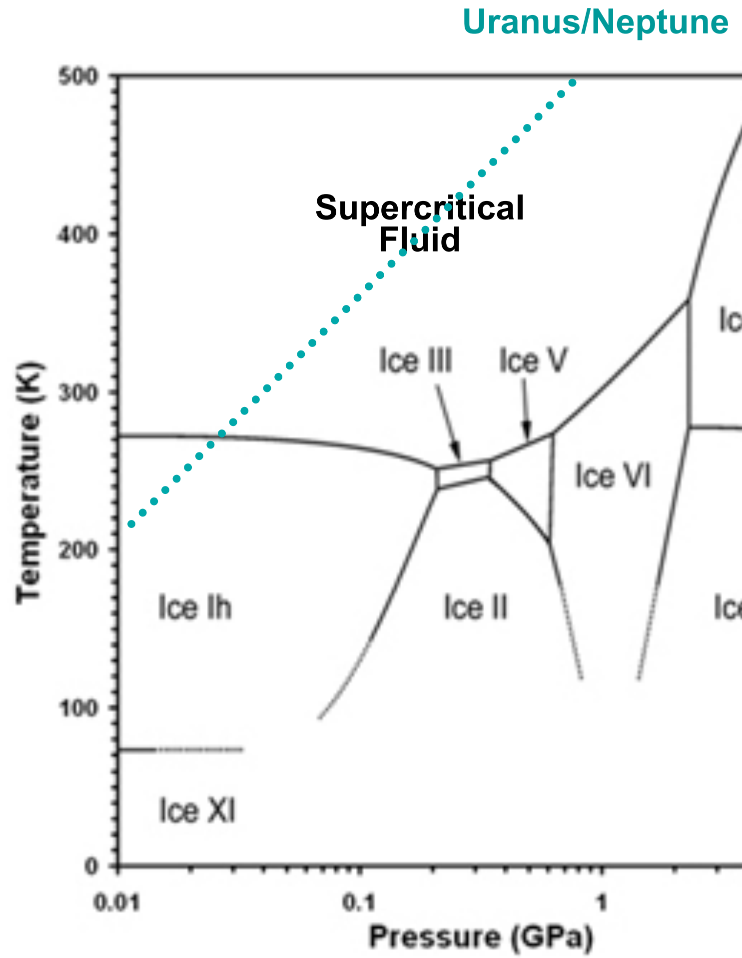


Phase Diagram for Hydrogen: Wikipedia

Material Properties

For ice giants like Uranus and Neptune, understanding how water and ammonia behave and interact is critically important for modeling their interiors.

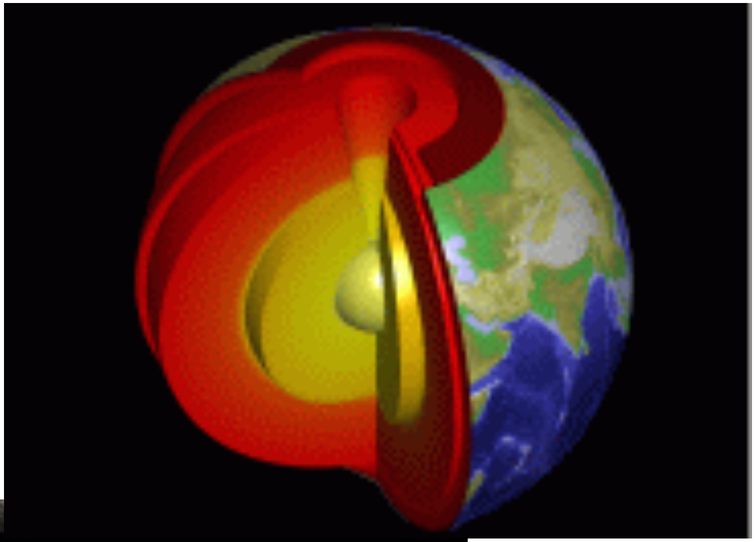
At high pressures, the liquid state of water becomes a supercritical fluid (i.e. not a true gas nor liquid) with higher conductivity

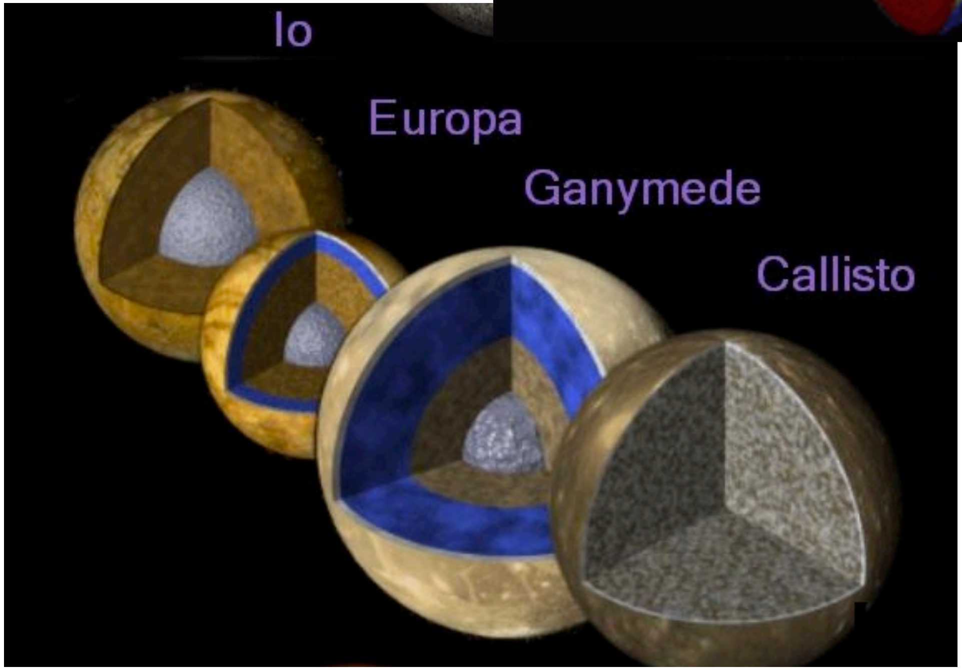


Phase Diagram for Water, Modified from UCL

Planetary Interiors

Earth's Interior Structure
Hydrostatic Equilibrium
Heating
Constituent Relations
Gravitational Fields
Isostasy
Magnetism





Gravity Fields

Gravitational Potential:

$$\Phi_g(r,\phi,\theta) = -\left(\frac{GM}{r} + \Delta\Phi_g(r,\phi,\theta)\right)$$

Where the first term represents a non-rotating fluid body in hydrostatic equilibrium, and the second term represents deviations from that idealized scenario.

In the most general case:

$$\Delta\Phi_g(r,\phi,\theta) = \frac{GM}{r}\sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(\frac{R}{r}\right)^n \left(C_{nm}\cos m\phi + S_{nm}\sin m\phi\right)P_{nm}(\cos\theta)$$

Where C_{nm} and S_{nm} are determined by internal mass distribution, and P_{nm} are the Legendre polynomials

Gravity Fields

$$\Delta\Phi_g(r,\phi,\theta) = \frac{GM}{r}\sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(\frac{R}{r}\right)^n \left(C_{nm}\cos m\phi + S_{nm}\sin m\phi\right)P_{nm}(\cos\theta)$$

This can be greatly simplified for giant planets with a few assumptions:

Axisymmetric about the rotation axis → S_{nm}, C_{nm}= 0 for m > 0 (reasonable since most departures from a sphere are due to the centrifugally driven equatorial bulge)

$$\Phi_g(r,\phi,\theta) = -\frac{Gm}{r}\left[1 - \sum_{n=2}^{\infty} J_n P_n(\cos\theta)\left(\frac{R}{r}\right)^n\right]. \qquad (2.29)$$

For N-S symmetry (not Mars!), also J_n=0 for odd n