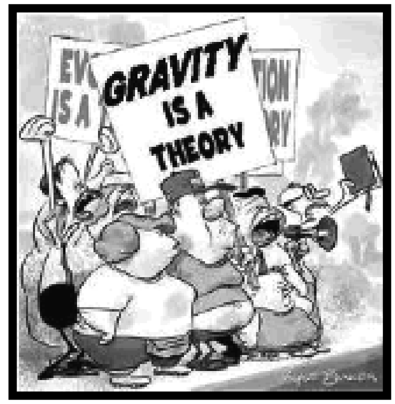
Homework 2: Due Wednesday 9/14/2016 in class



Required Reading:
Chapters 1-2 should be completed!

Questions:
For this assignment, **everyone** should answer the questions below from the textbook:

Chapter 1: 1.6

Chapter 2: 2.3, 2.4, 2.33 **Graduate Students Only:** 2.26

Also, **everyone** should complete the following question:

Tidal evolution can limit the possible existence of sub-satellites (satellites orbiting satellites). Consider a satellite of mass m_s orbiting a planet of mass M at a distance a_s with mean motion n_s. Now examine the stability of a smaller body (m_{ss}) orbiting the satellite at a distance a_{ss} and mean motion n_{ss}.

a. By comparing the tidal force exerted by the planet across the orbital radius of m_{ss} with the gravitational attraction of m_{ss} by the satellite, **show** that the sub-satellite is only likely to remain in a stable keplerian orbit if:

$$a_{ss} \leq \left(\frac{m_s}{2M}\right)^{\frac{1}{3}} a_s$$

What will happen to the sub-satellite at larger distances?

b. A more rigorous calculation yields the upper limit:

$$a_{ss} \leq r_H = \left(\frac{m_s}{3M}\right)^{\frac{1}{3}} a_s$$

This is known as the Hill sphere radius of m_s in the 3-body problem.
Evaluate r_H for Io and for Titan.

c. Assuming that the satellite is in synchronous rotation (i.e. $\omega = n_s$), **show** that the radius of the synchronous orbit about m_s (where $n_{ss} = \omega$) is given by:

$$a_{syn} = \left(\frac{m_s}{M}\right)^{\frac{1}{3}} a_s$$

Given that $a_{syn} > r_H$, ***explain*** why the tides raised by the sub-satellite on the satellite will lead to the orbital decay and eventual loss of any such sub-satellite.

d. Can a similar argument be made for the absence of any satellites of Mercury and Venus? Consider the actual rotation rates of these planets (asynchronous relative to their solar orbits) in your answer, quantitatively if possible.