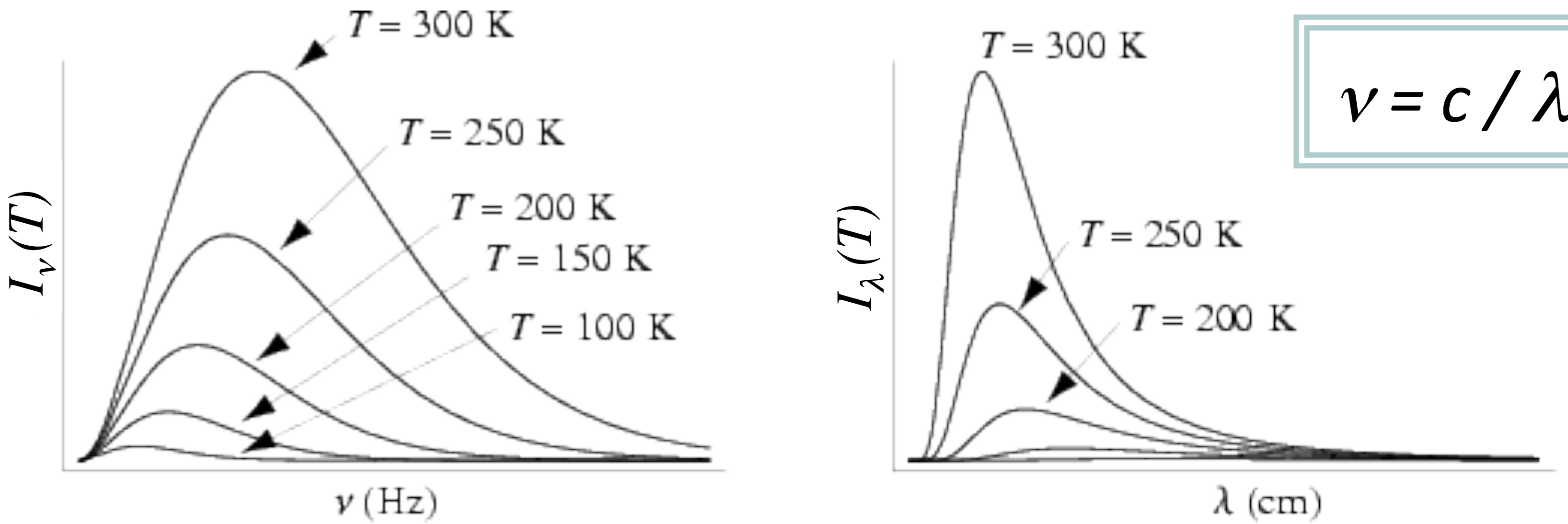
Planck's Law for Black Body Radiation



$$\nu = c / \lambda$$

$$I(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

Figure modified from Eric W. Weisstein

Specific Brightness:

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

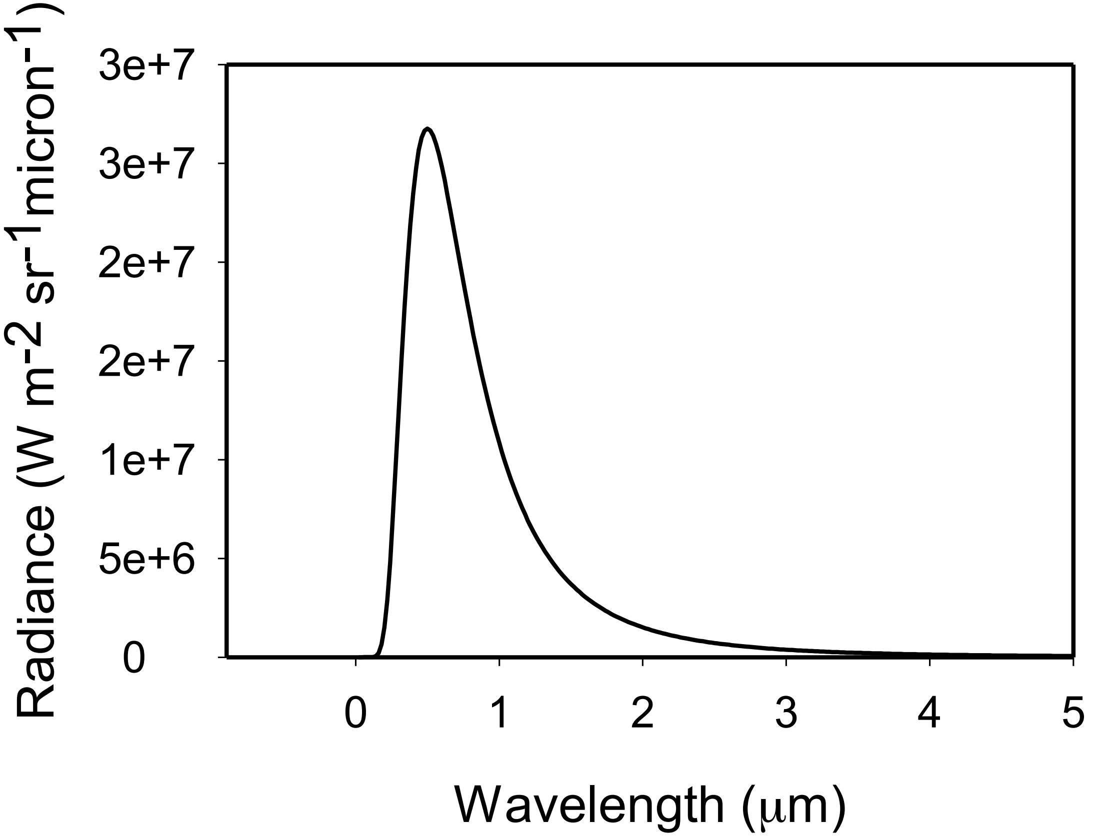
Properties of Blackbody Curves

- They don’t cross each other
- The wavelength of the maximum spectral radiance is inversely proportional to temperature – Wien’s Law:

For λ measured in microns and T in K:

$$\lambda_{\max} = 2898/T$$

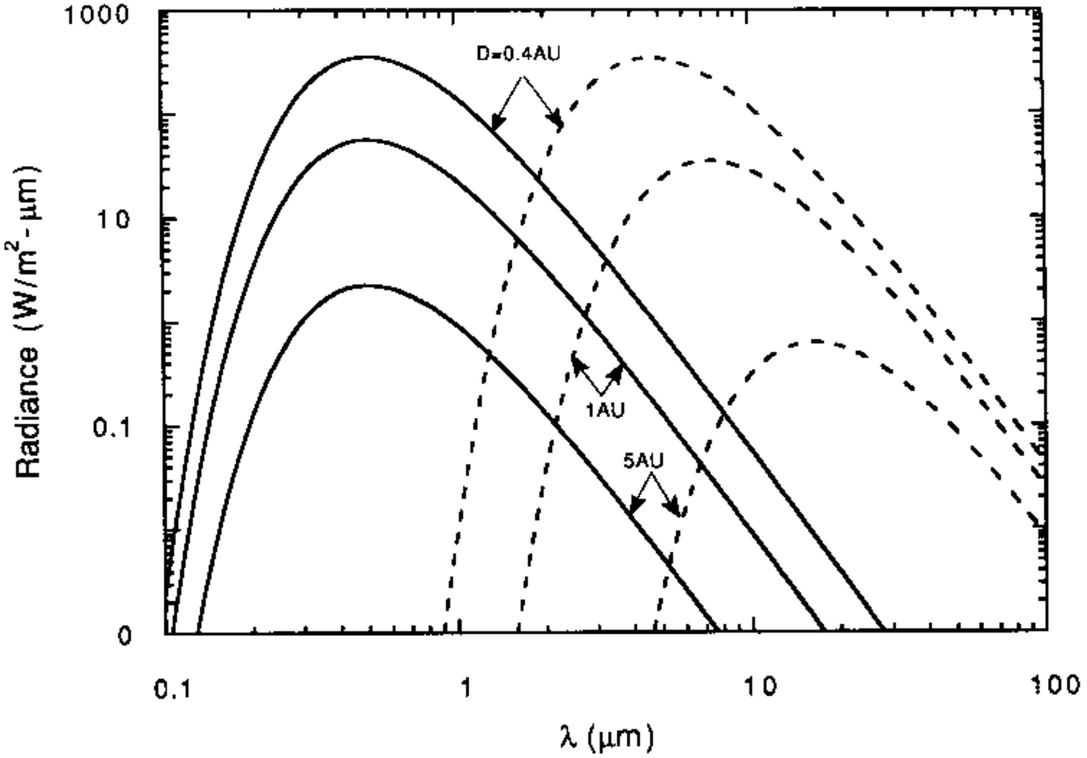
The Sun's spectrum is approximated as a Blackbody at 5800K



See Java tool at:
http://webphysics.davidson.edu/Applets/Blackbody/BlackBody.html

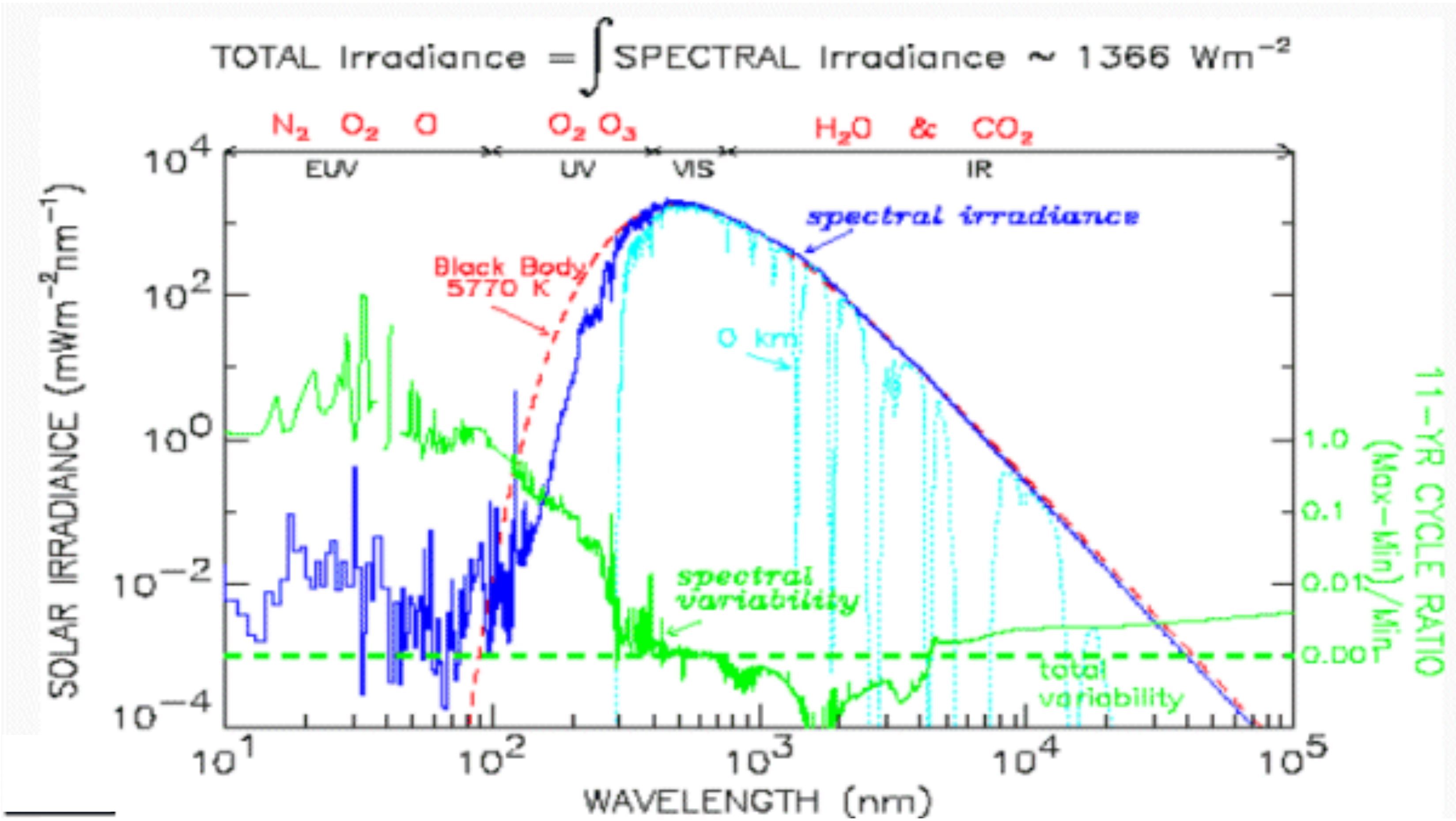
Solar radiance vs. Emission

Figure 13.1. Comparison of sunlight reflected (solid lines) from a surface with a visual albedo of 0.1 with the radiation thermally emitted (dashed lines) from the surface with an IR emissivity of 1.00, for three different distances from the sun.

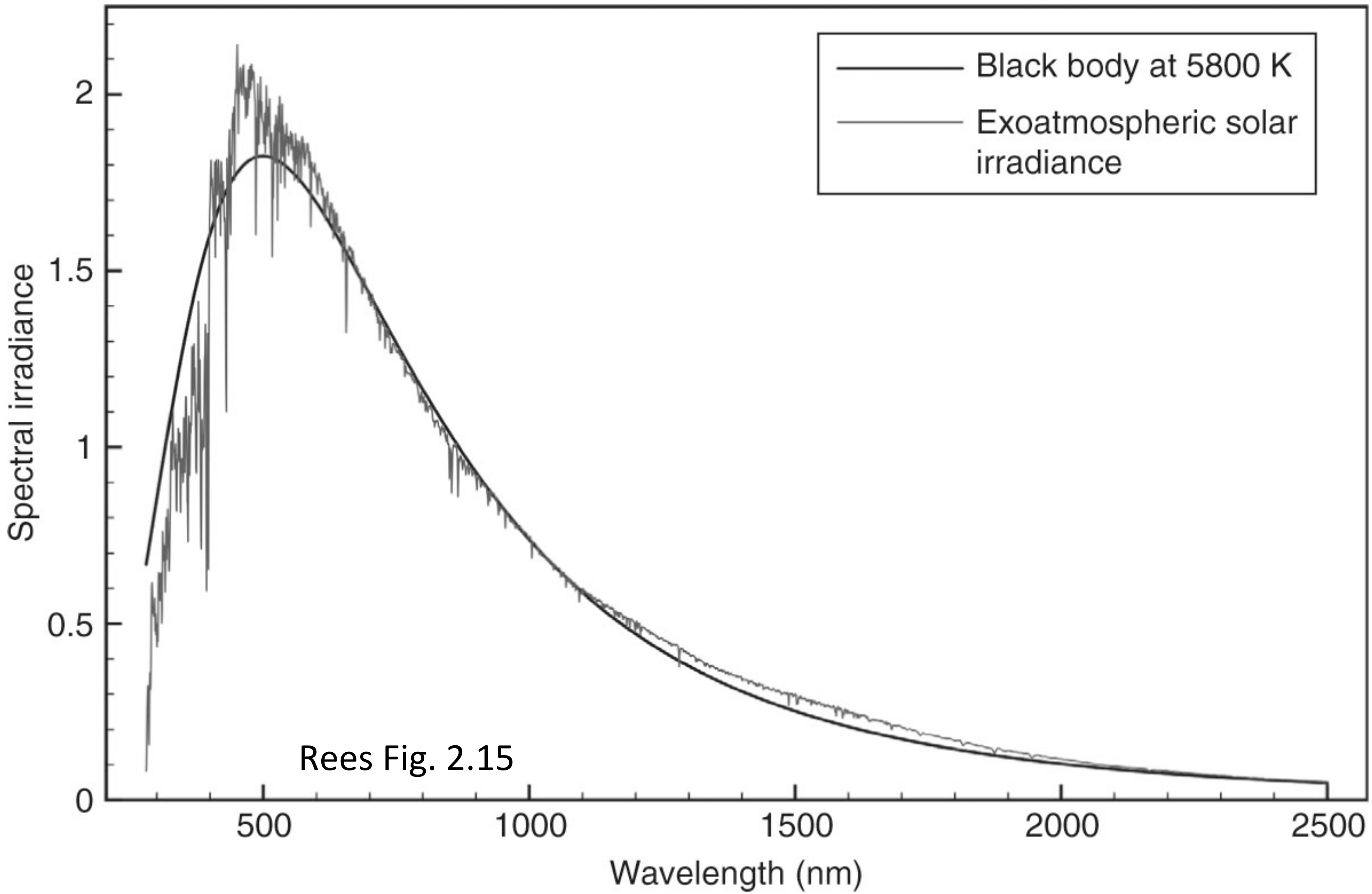


from Hapke, 1993

Solar Spectrum, Variability, and Atmospheric Absorption



A linear plot of the solar spectrum



Rees Fig. 2.15

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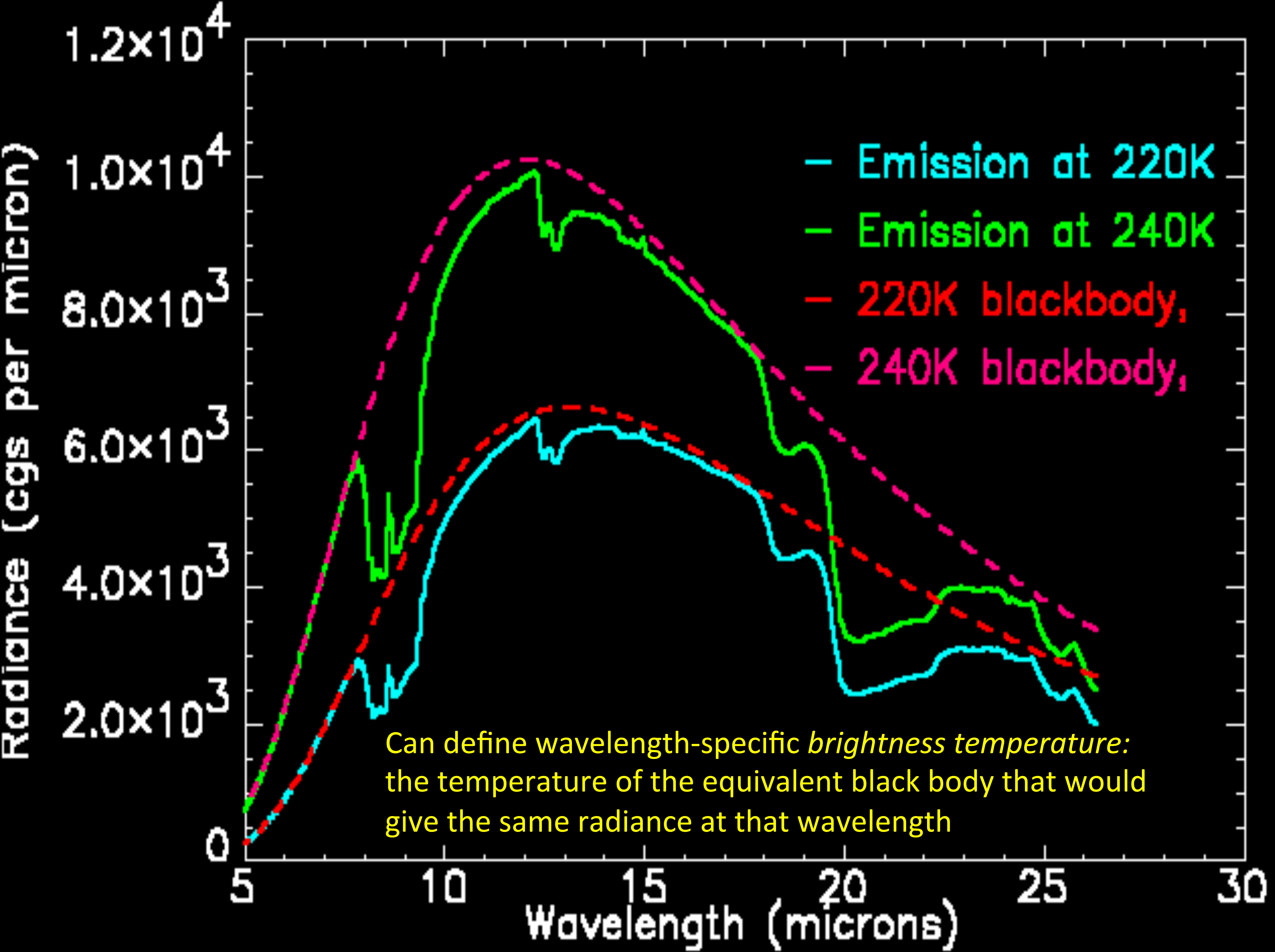
Emission and Reflection

- Most “real” materials do not emit as perfect blackbodies.

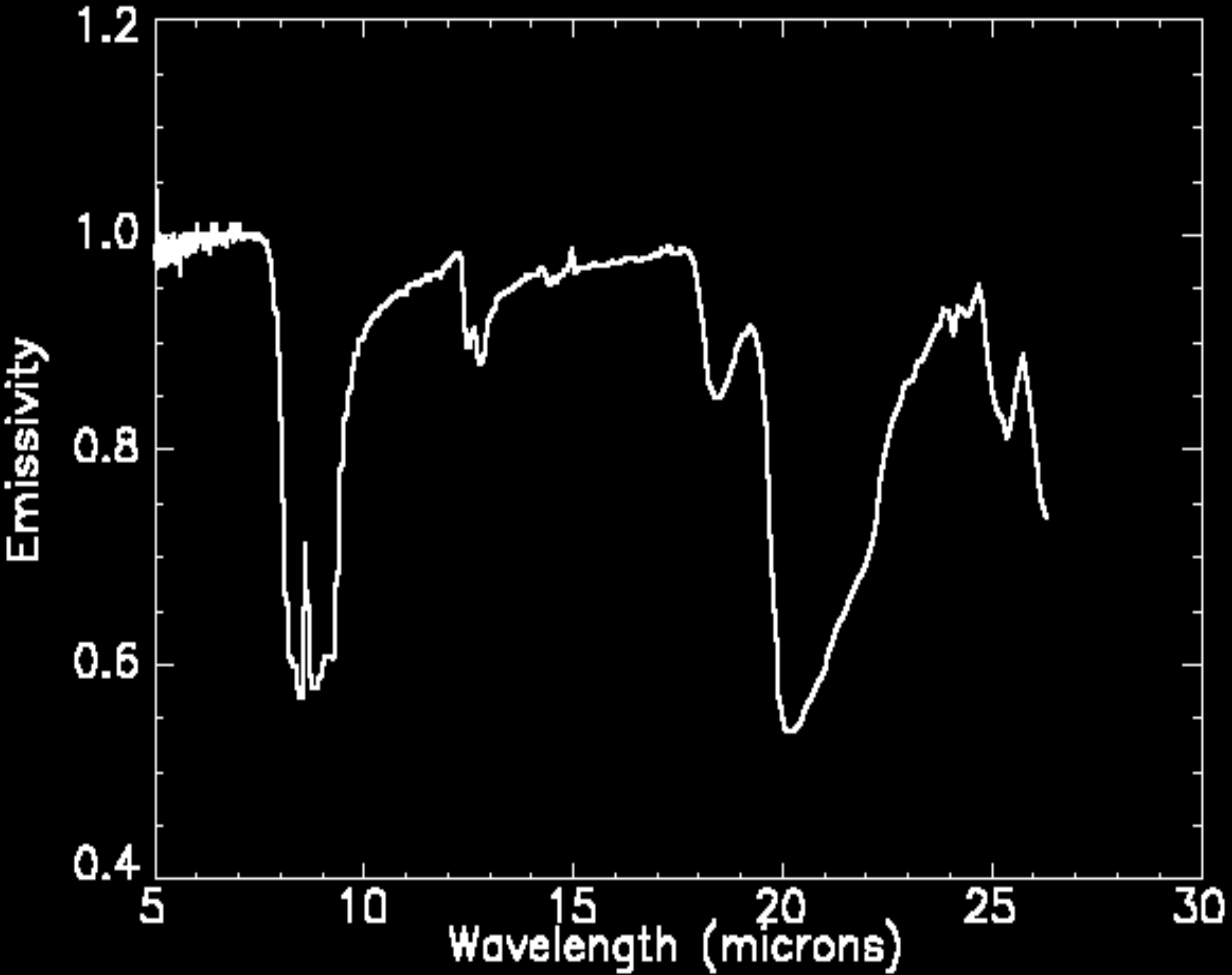
  Emissivity (ε) is the ratio of (radiation actually emitted) to (the radiation of a blackbody at the same temperature). Emissivity varies with wavelength.

$$\varepsilon(\lambda) = L(\lambda, T) / L_{bb\_emit}(\lambda, T)$$

$$L(\lambda, T) = \varepsilon(\lambda)\, L_{bb\_emit}(\lambda, T)$$



Can define wavelength-specific *brightness temperature:* the temperature of the equivalent black body that would give the same radiance at that wavelength


1.2
1.0
0.8
0.6
0.4
Emissivity
5
10
15
20
25
30
Wavelength (microns)

Emission and Reflection

- Likewise, most materials do not reflect perfectly.

  Reflectance (R) is the ratio of (radiation reflected) to (the incident radiance).

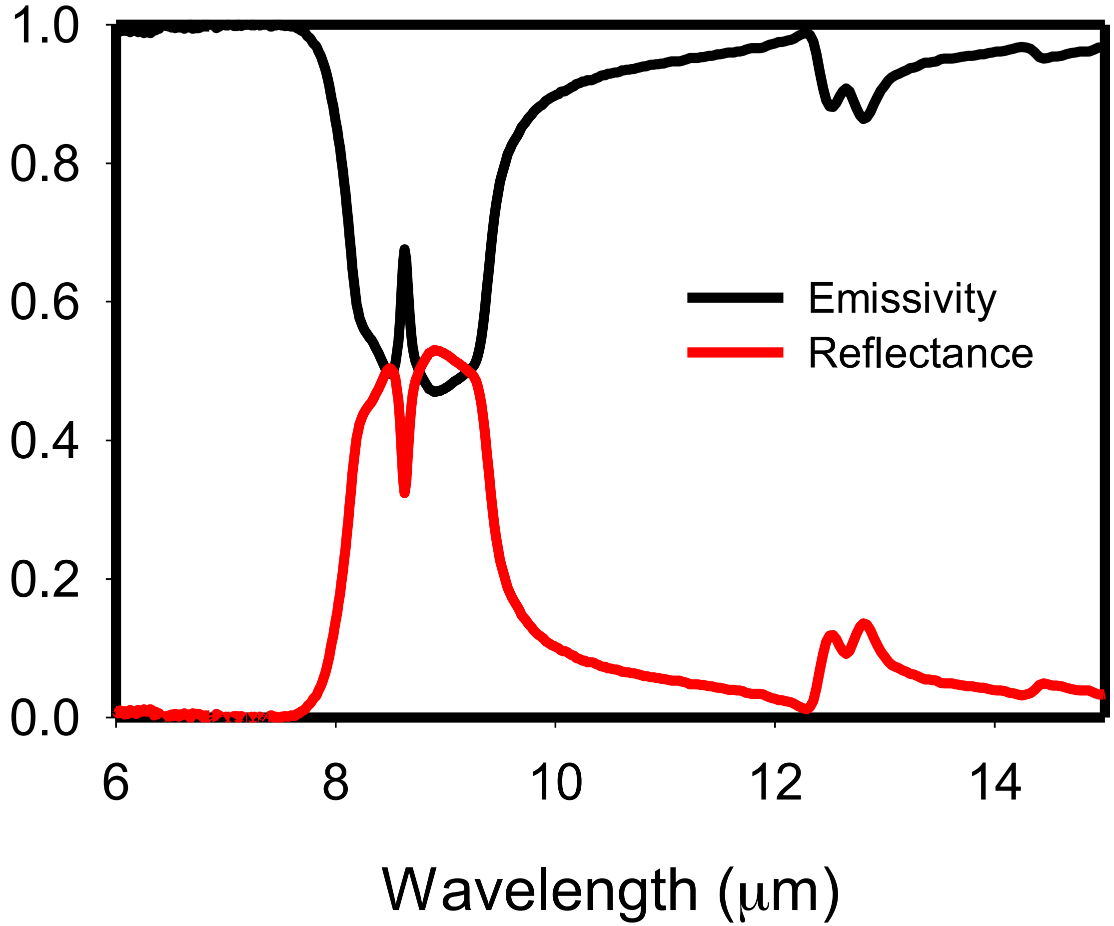
$$R(\lambda) = L(\lambda,T) \,/\, L_{bb\_incid}(\lambda,T)$$

$$L(\lambda,T) = R(\lambda)\, L_{bb\_incid}(\lambda,T)$$

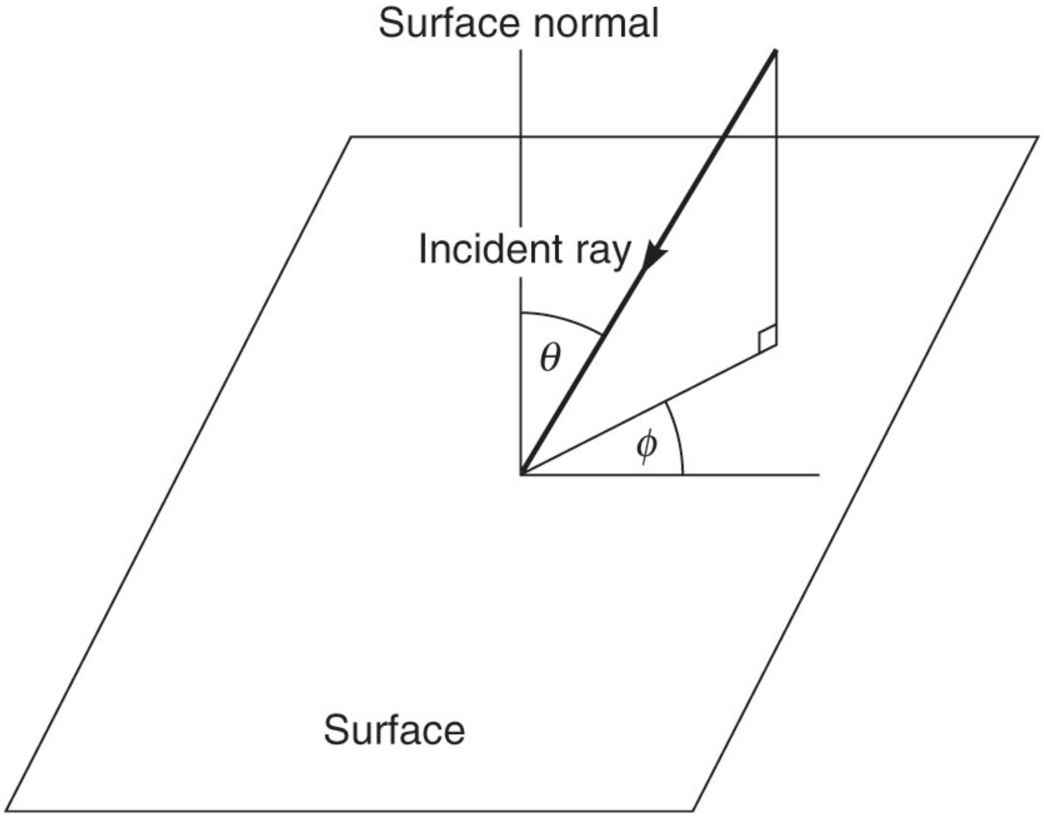
Kirchhoff's Law

$\varepsilon = 1 - R$

Empirically verified to high precision



Angular distributions of radiation



Rees Fig. 2.9

Incidence angle θ, azimuthal angle ϕ

*can also define **emergence angle**,*

*and **phase angle** between incidence and emergence*

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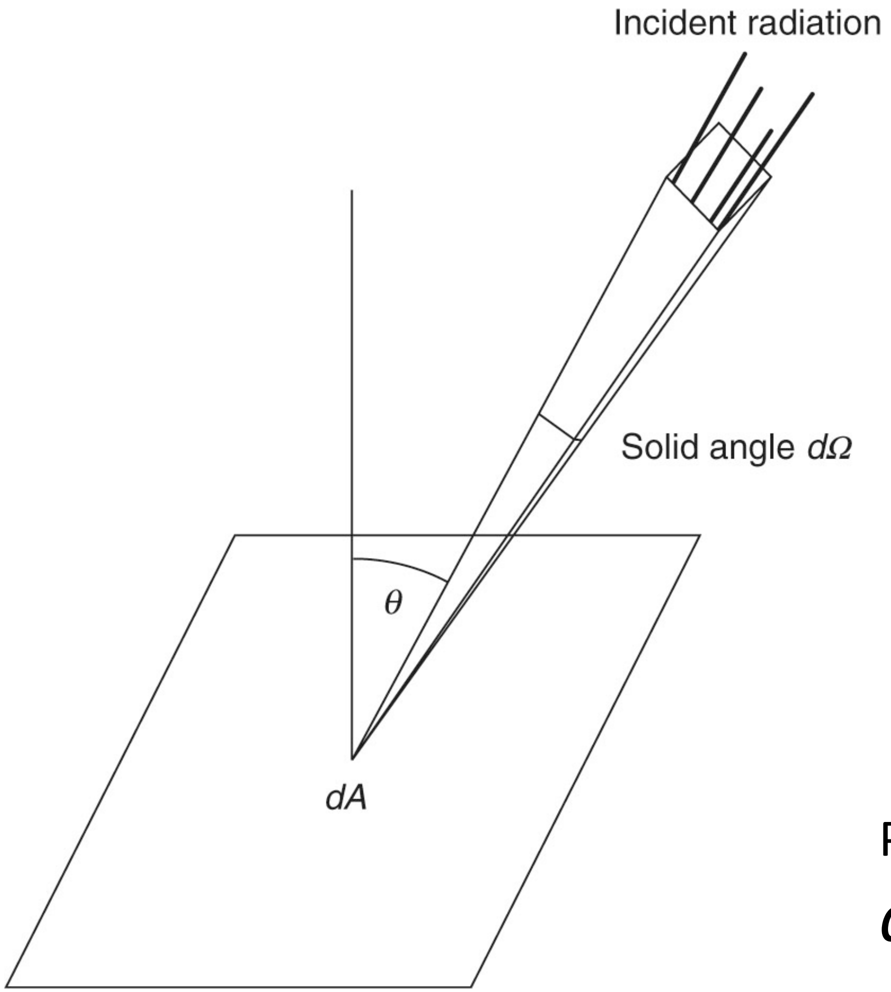
Angular distributions of radiation



Rees Fig. 2.10

$d\Omega = \sin\theta d\theta d\phi$

Measured in steradians, of which there are 4π in 360°

Power incident on *dA* from *dΩ*:

$dP = L\cos\theta dA d\Omega$

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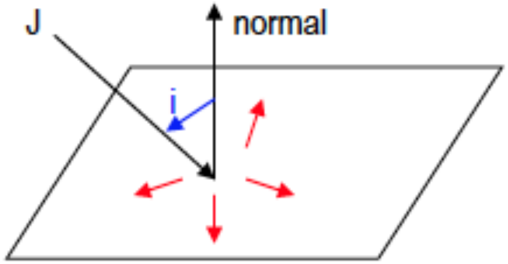
Lambert Albedo

$$J\cos(i)A_L = \pi I$$

Incident irradiance, collimated beam

Lambert Albedo

I integrated over outgoing hemisphere



A_L is a directional, hemispherical albedo

$$A_L = \pi I / J\cos(i)$$

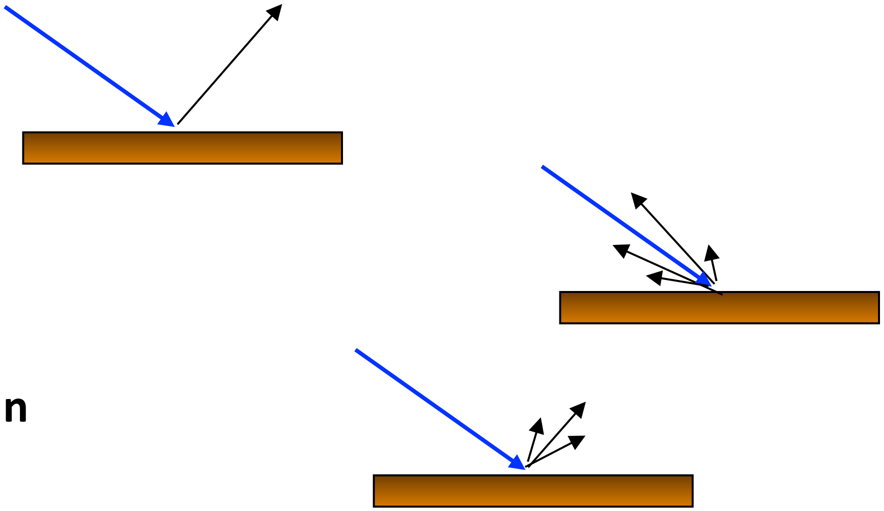
Note that I does not vary with emergence or phase angles only the incidence angle.

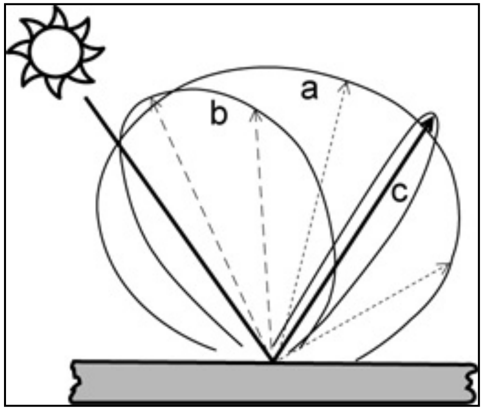
Makes it relatively easy to model the surface reflectance and retrieve Lambert Albedos

Not a bad assumption for modest values of incidence, emergence, and phase angles

Surfaces may be

- **specular**
- **back-reflecting**
- **forward-reflecting**
- **diffuse or Lambertian**





Reflection envelopes

Smooth surfaces (rms<<λ) generally are specular or forward-reflecting
examples: water, ice

Rough surfaces (rms>>λ) generally are diffuse
example: sand

Complex surfaces with smooth facets at a variety of orientations are forward- or back-reflecting
example: leaves

Interaction of light with materials

relative electric permittivity $\varepsilon_r = \varepsilon/\varepsilon_0$

a.k.a. ***dielectric constant***

(not to be confused with emissivity…)

For magnetic materials:

relative magnetic permeability $\mu_r = \mu/\mu_0$

Updated solution to Maxwell's equations:

$E_z = E_0\cos(\omega t - ky);$ $E_x = E_y = 0$

$B_x = (E_0\surd(\varepsilon_r\mu_r)/c)\cos(\omega t - ky);$ $B_y = B_z = 0$

phase velocity $v = \omega/k = c/\surd(\varepsilon_r\mu_r) = c/n$*,*
where n *is the* ***refractive index***

Interaction of light with materials

Absorptive materials have a *complex* dielectric constant

$$\varepsilon_r = \varepsilon' - i\varepsilon''$$

$= \varepsilon'(1 - \mathrm{i}\tan\theta)$ for ***loss tangent*** $\tan\theta$

This, in turn, makes *n* complex …

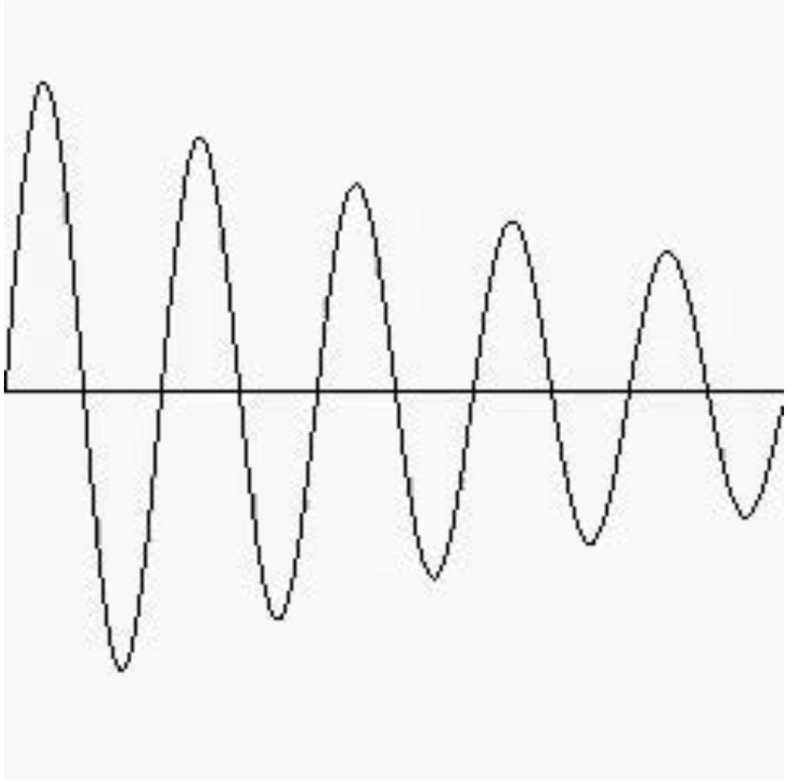
The Index of Refraction

- Index of refraction is a complex quantity that is wavelength dependent

$$m(\lambda) = n(\lambda) - ik(\lambda)$$

  - Real portion "n": index of refraction
  - Imaginary portion "k": absorptivity
    - Also called "optical constants"
  - Wavelength (λ) dependence is important: this is why spectroscopy works!

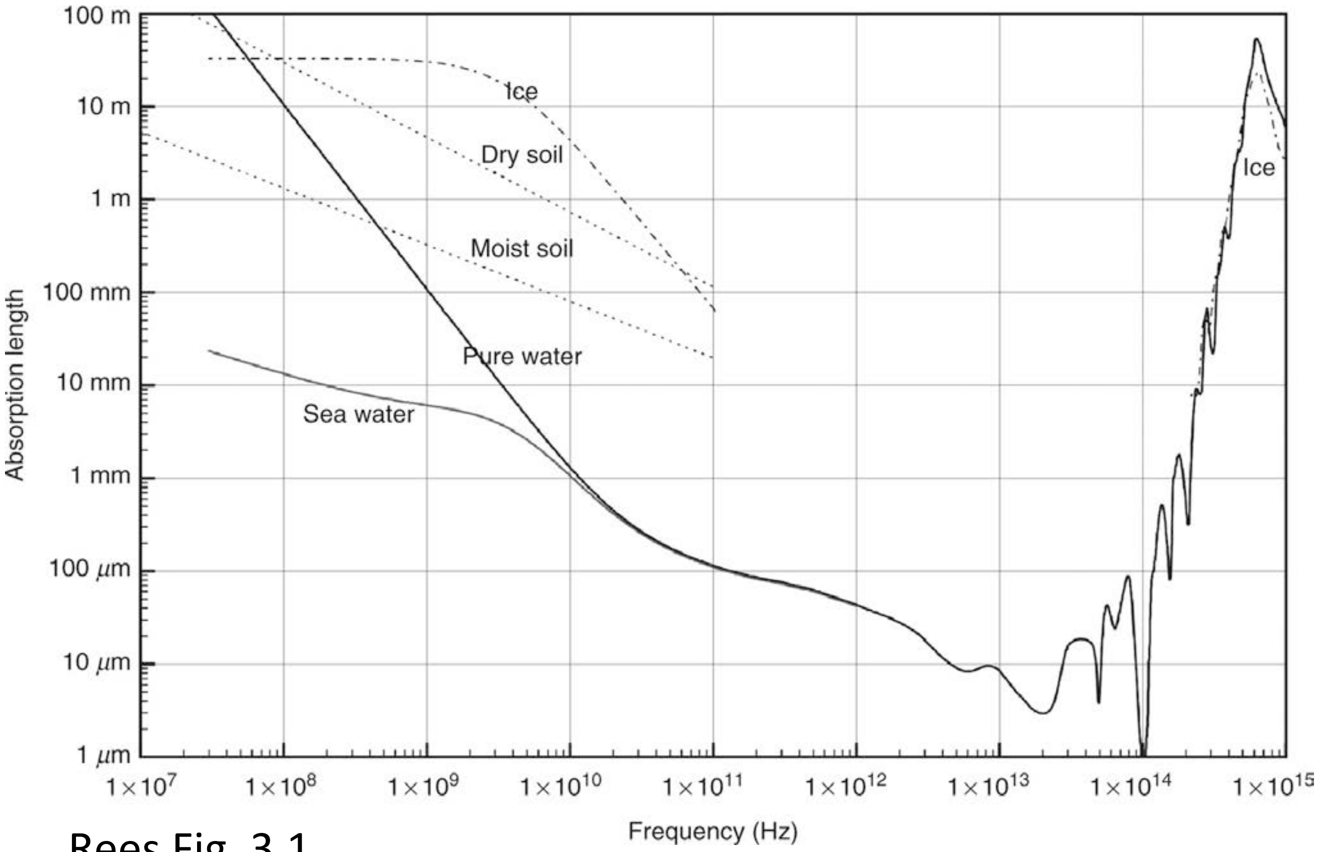
Absorptive media



For a gas that is not too strongly absorbing, $n \approx 1 + N\alpha/2\varepsilon_0$,
for N = *# density of gas molecules*, α = *polarisability*

Absorptive materials

Wave amplitude decreases exponentially with propagation distance
After one “absorption length” l_a, is reduced by factor $1/e$



Rees Fig. 3.1

Q: where is the visible/infrared range on this diagram?

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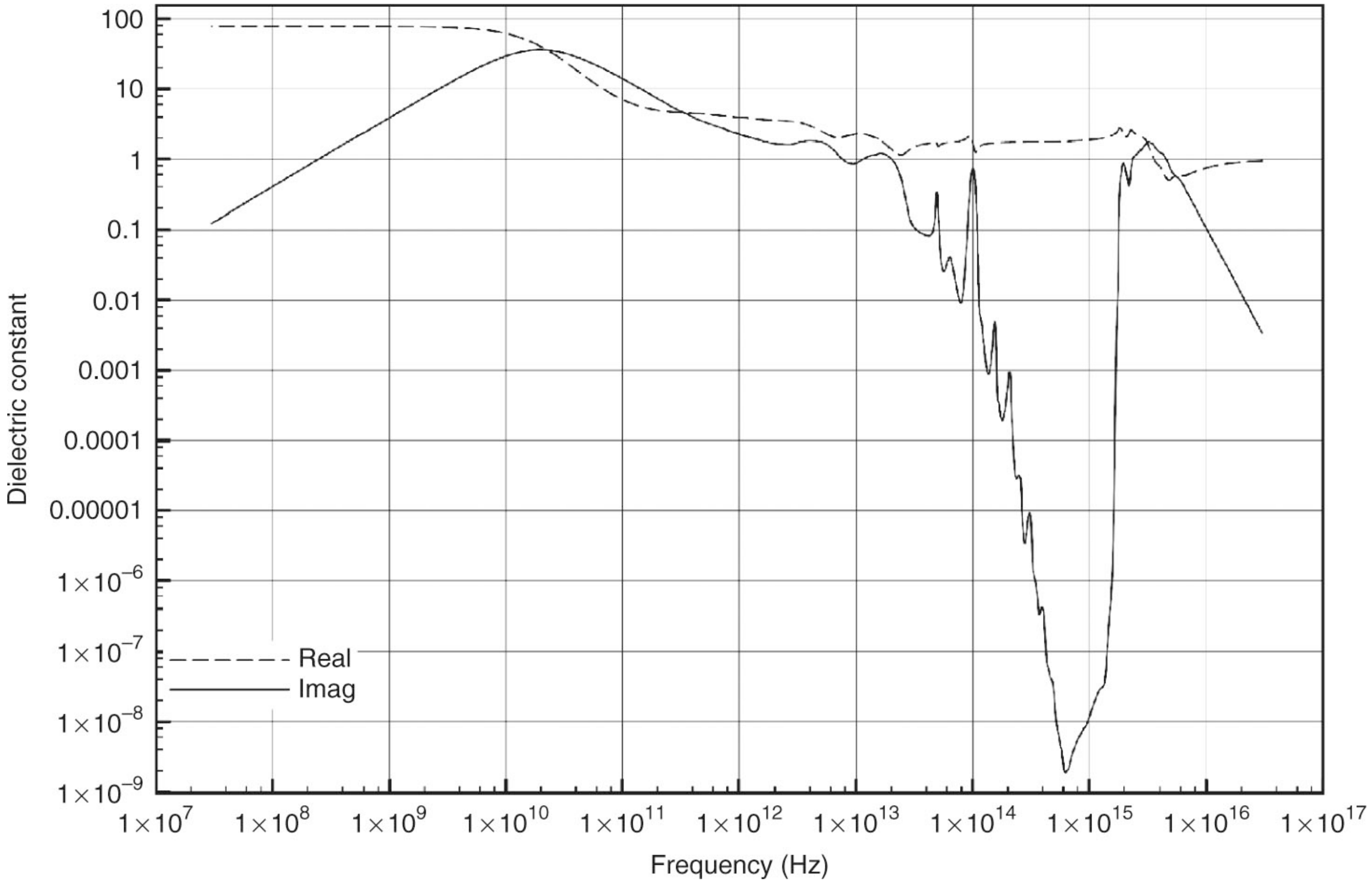
Dielectric constant of pure water



Rees Fig 3.2

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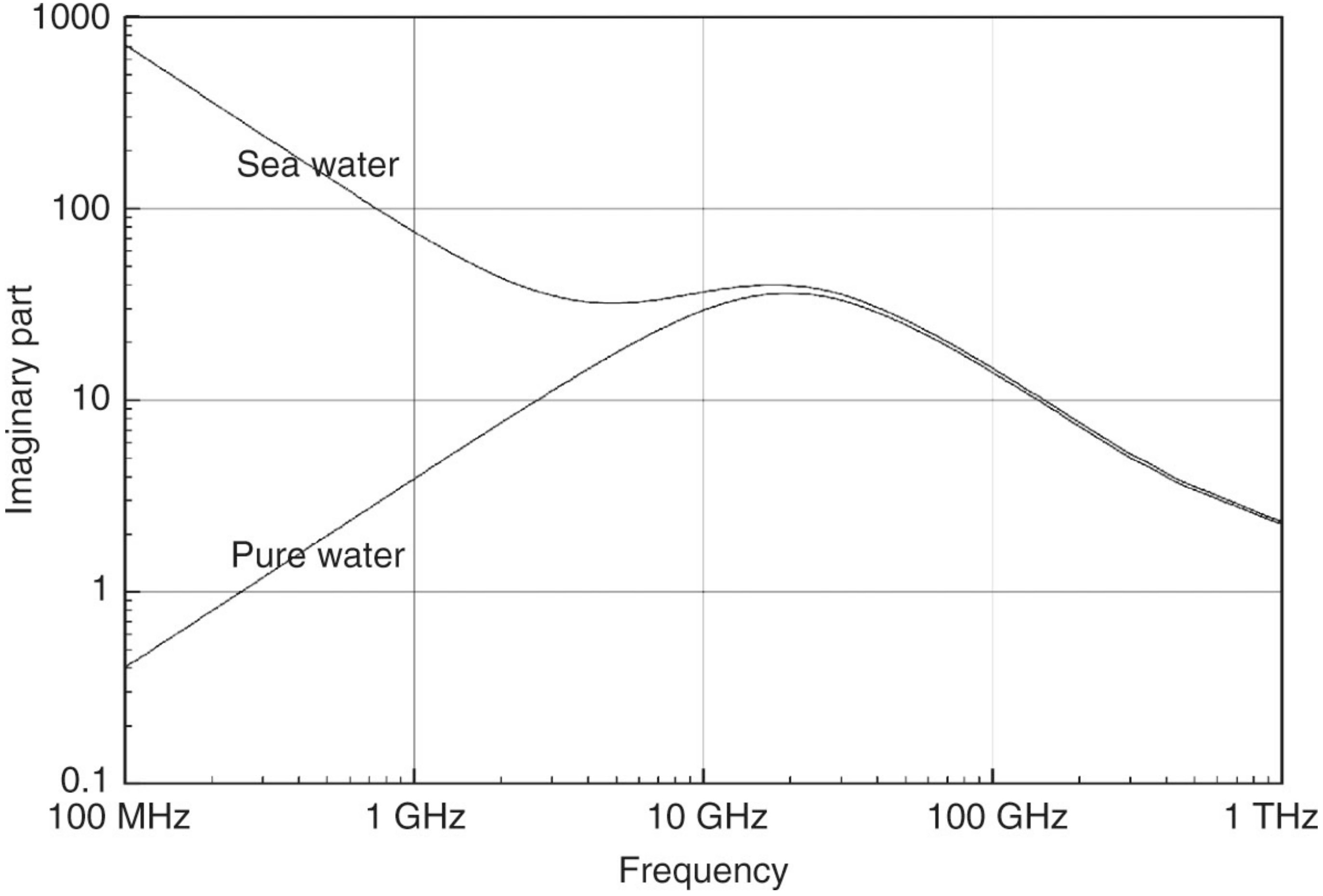
Dielectric constant: pure water vs. sea water



Rees Fig 3.3

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Quartz, n

Quartz, k