The Index of Refraction

- Index of refraction is a complex quantity that is wavelength dependent

$$m(\lambda) = n(\lambda) - ik(\lambda)$$

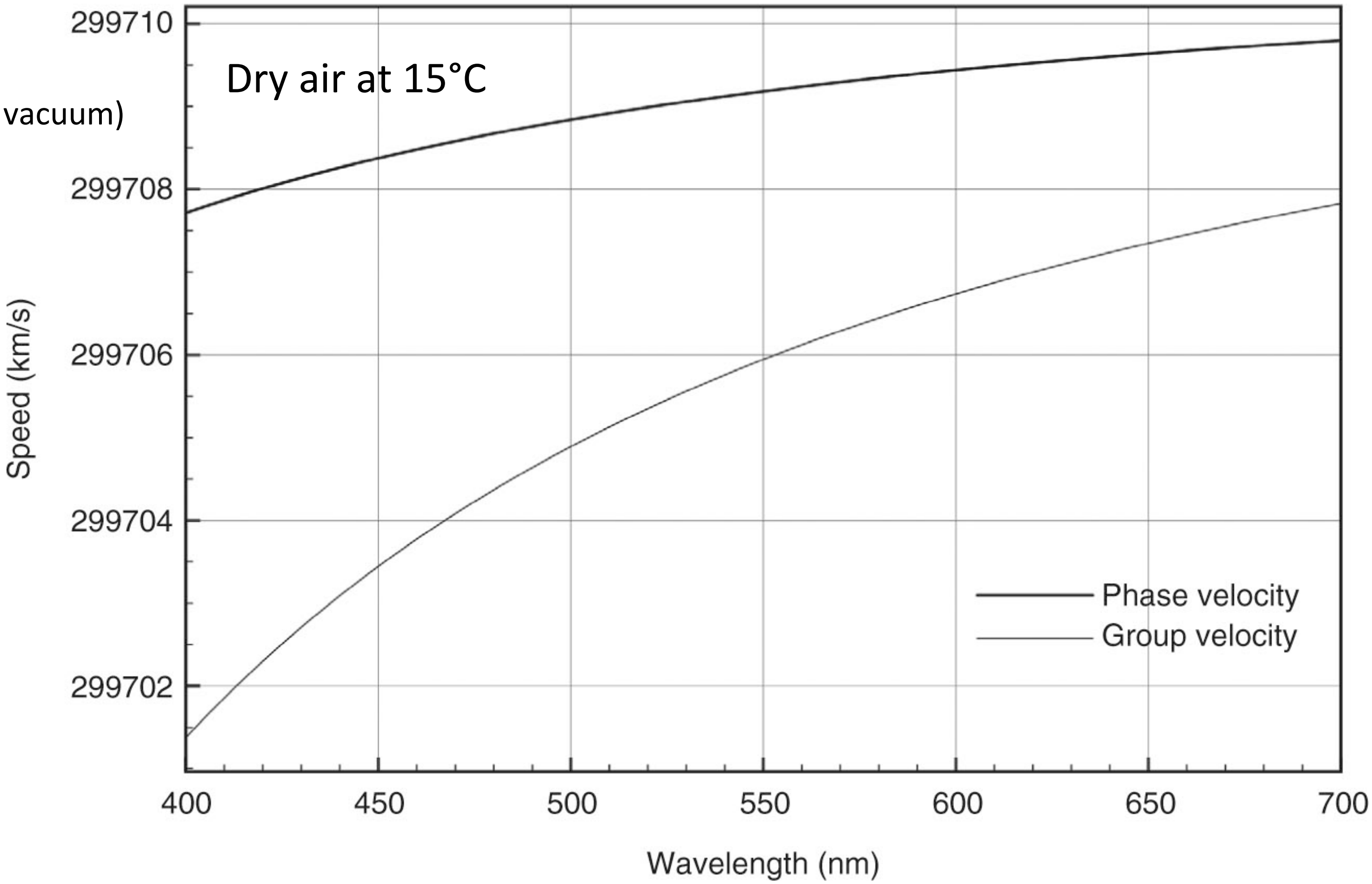
  - Real portion "n": index of refraction
  - Imaginary portion "k": absorptivity
    - Also called "optical constants"
  - Wavelength (λ) dependence is important: this is why spectroscopy works!

Dispersion

If n varies with wavelength, then so does phase velocity v = ω/k = c/n,

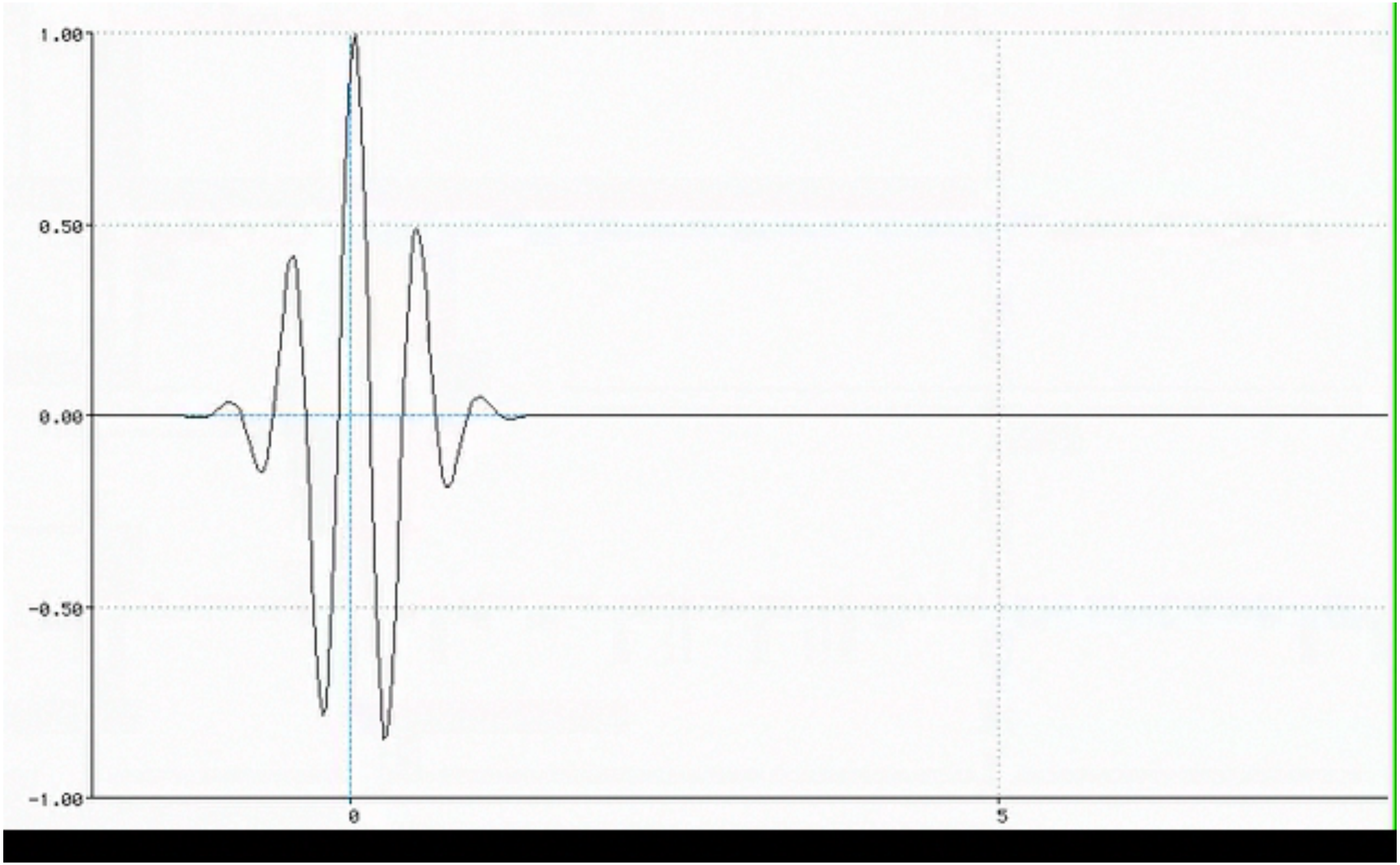
… and ***group velocity*** *$v_g = d\omega/dk$*

(299792 in vacuum)



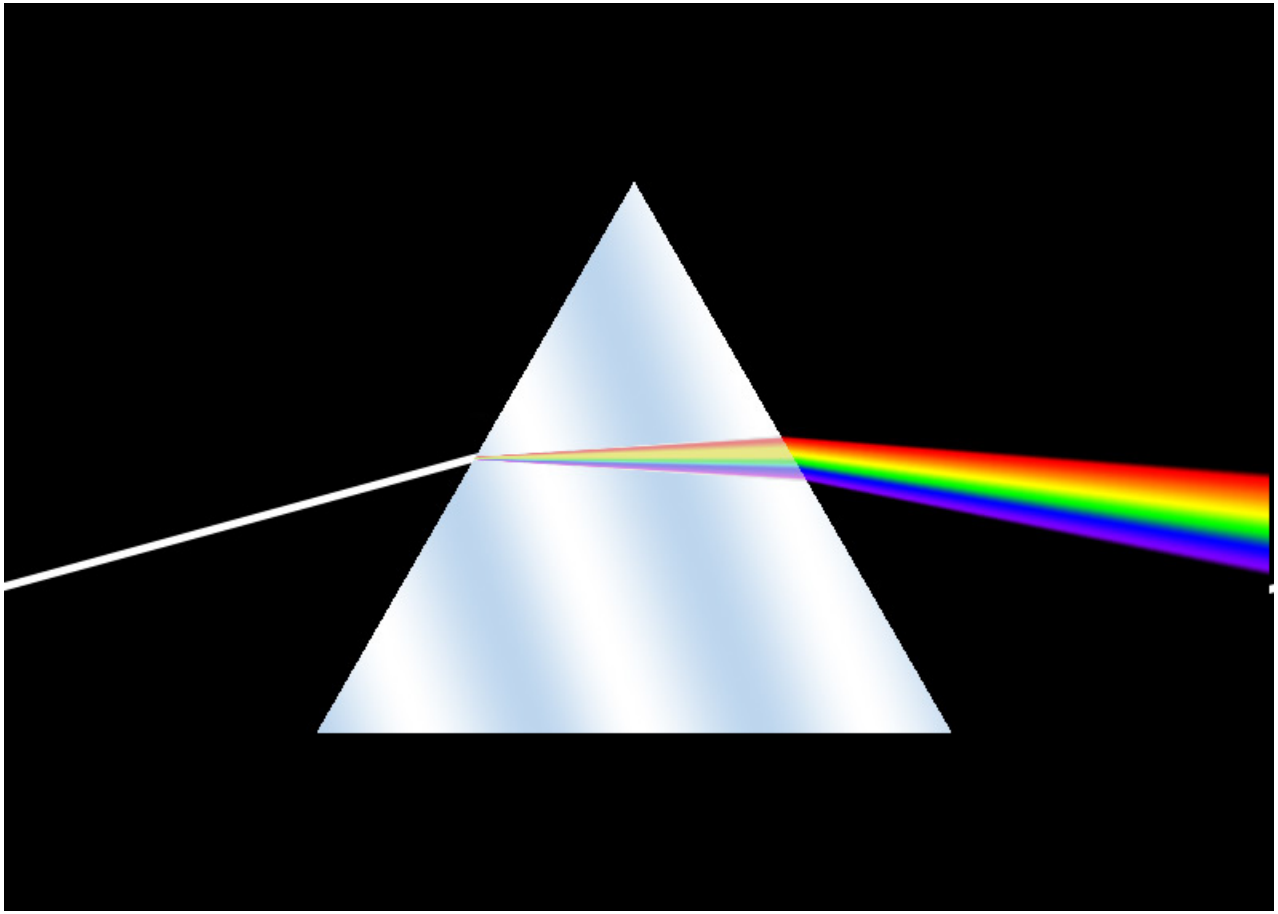
Rees Fig. 3.5

Phase velocity vs. group velocity

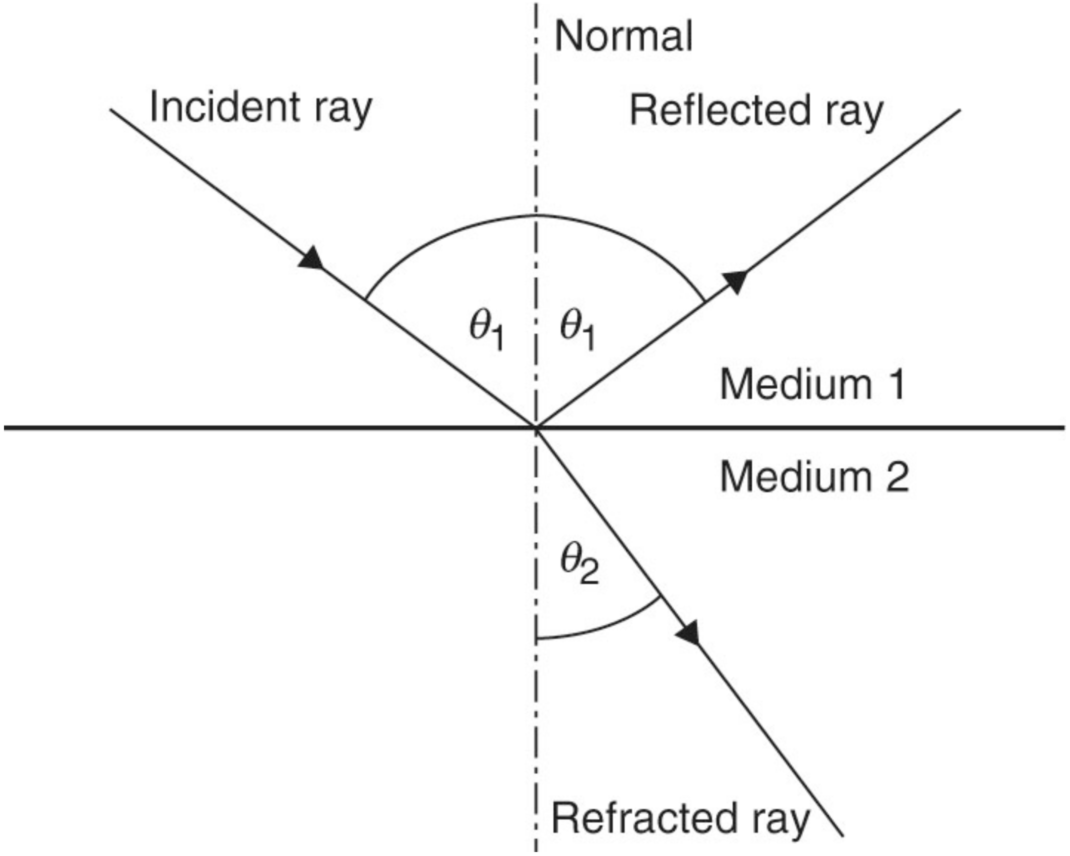


[Which is larger here?]

Refraction through a prism: n, k are functions of λ



Transmitted light is ***refracted***

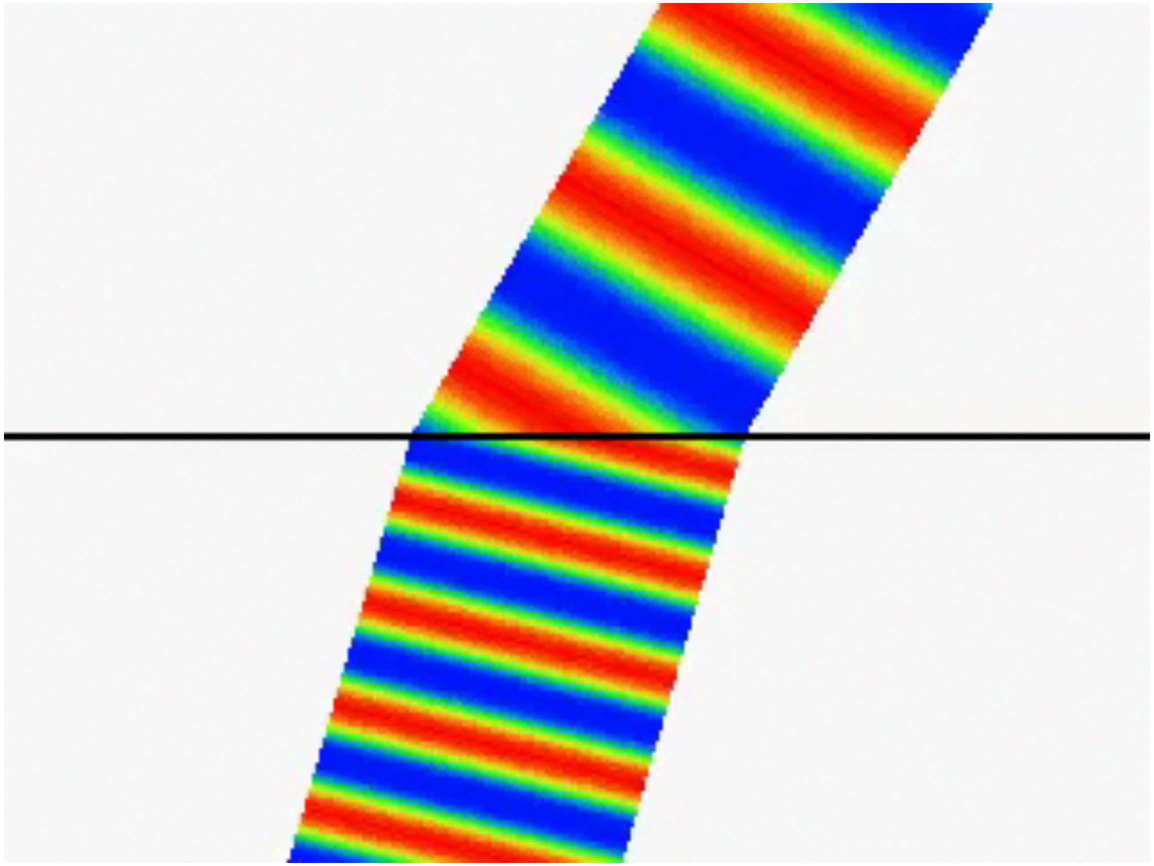


Rees Fig. 3.6

<u>Snell's law:</u>

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Snell’s law



Consider how refraction can distort your perception of distance…

Fresnel Reflectance

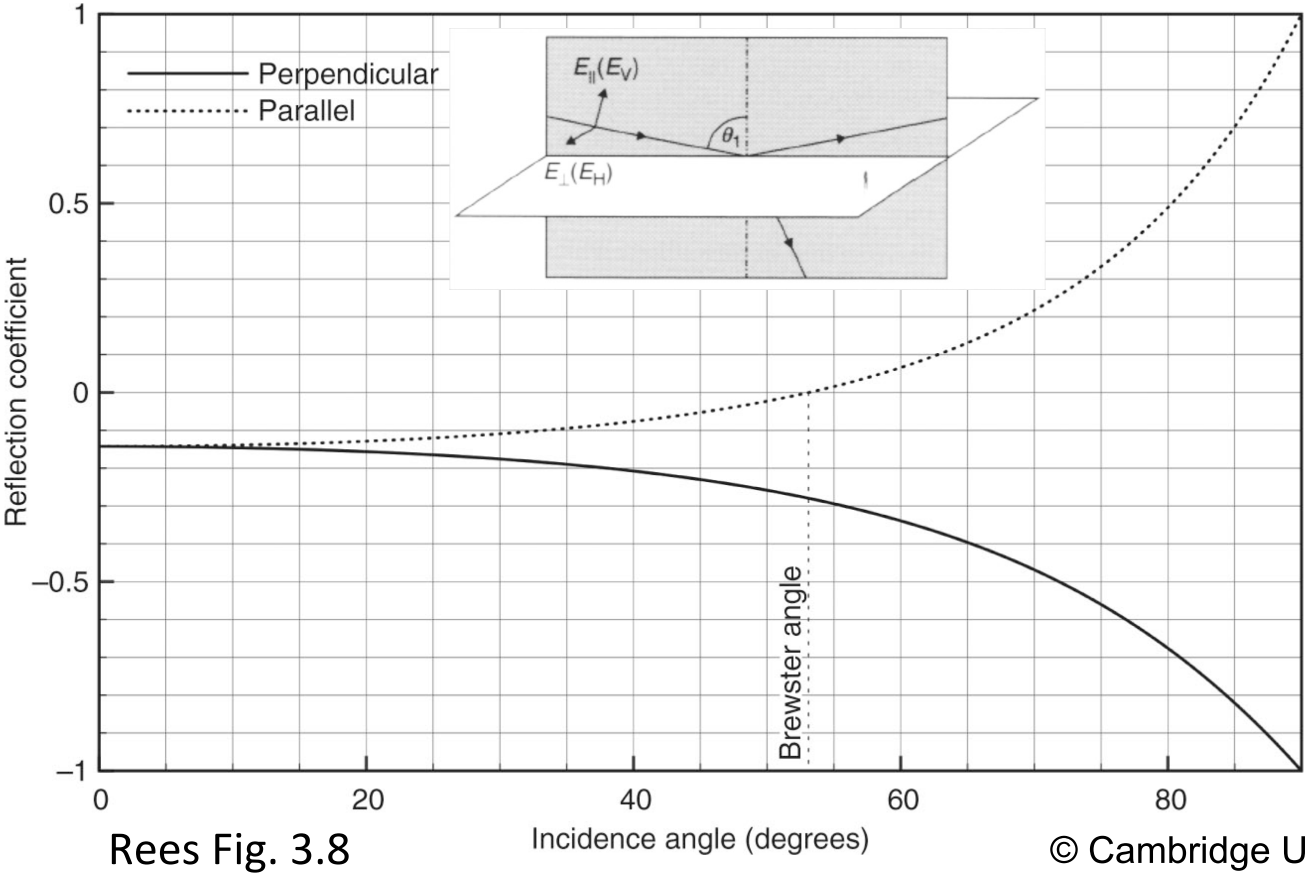
- Fresnel equations describe reflectance from a specular (smooth) surface in terms of the optical constants
  - In the special case of normal incidence:

$$R = \frac{(n-1)^2 + k^2}{(n+1)^2 + k^2}$$

Note: All quantities above are functions of λ.

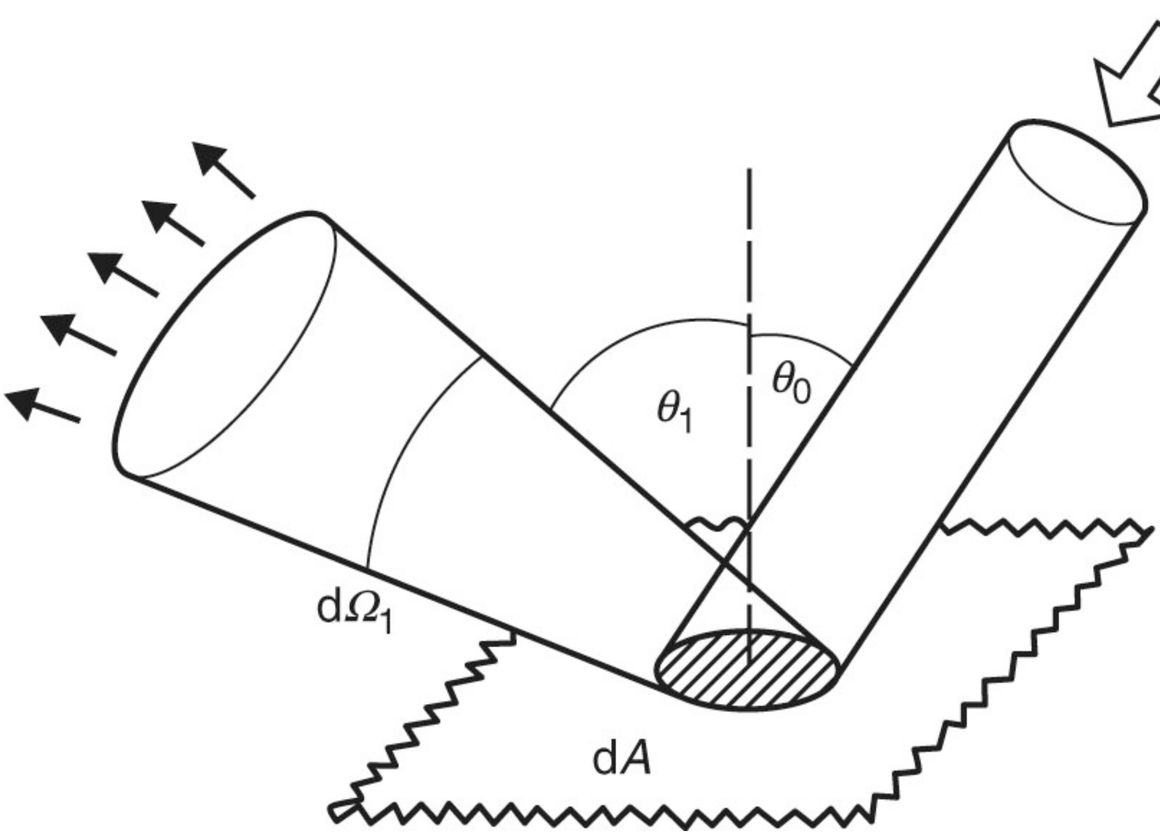
Fresnel's equations…

are actually much more complicated, generally giving the reflection, transmission coefficients as functions of θ_1, θ_2, polarization



Rees Fig. 3.8

Scattering

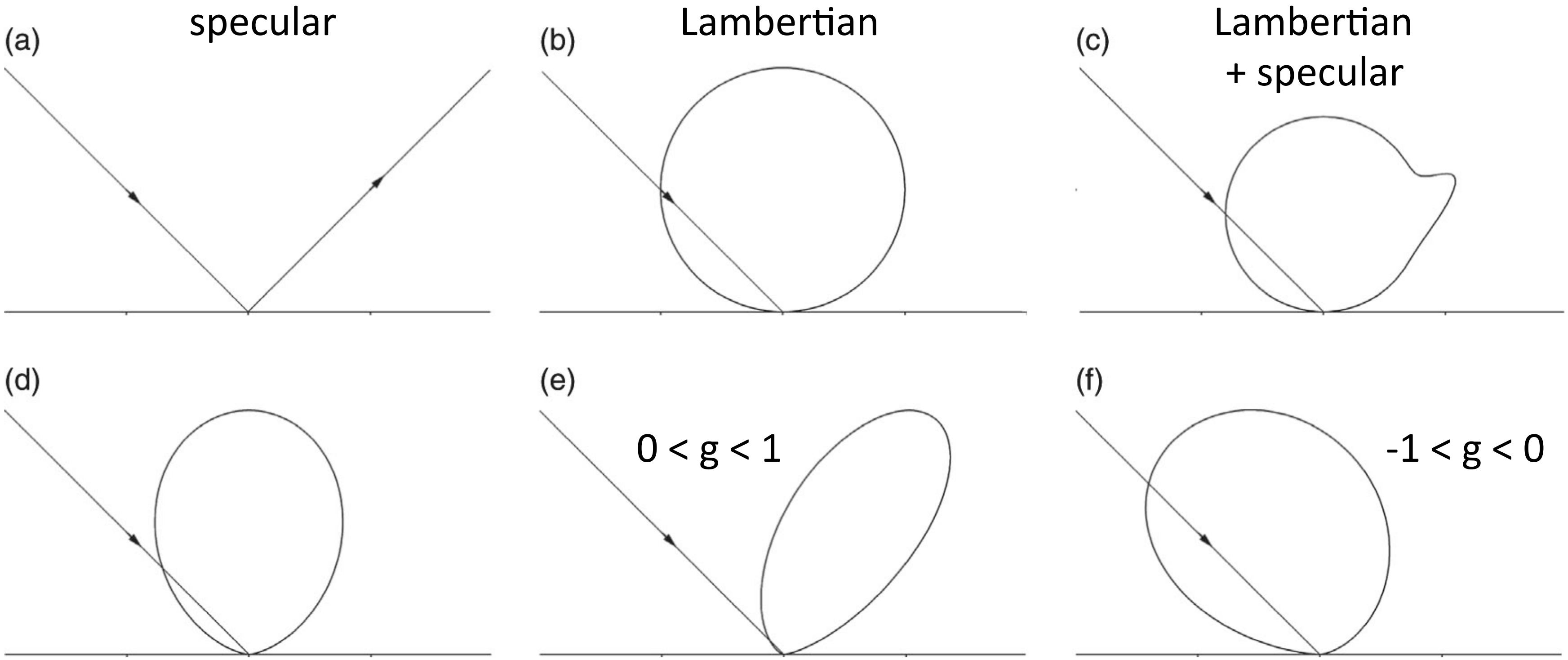


Rees Fig. 3.9

BRDF

Q: What irradiance is measured at the surface?
A: $F\cos\theta_0$

Reflection envelopes



specular

Lambertian

Lambertian + specular

$0 < g < 1$

$-1 < g < 0$

Minnaert

BRDF ~ $(\cos\theta_0\cos\theta_1)^{\kappa-1}$

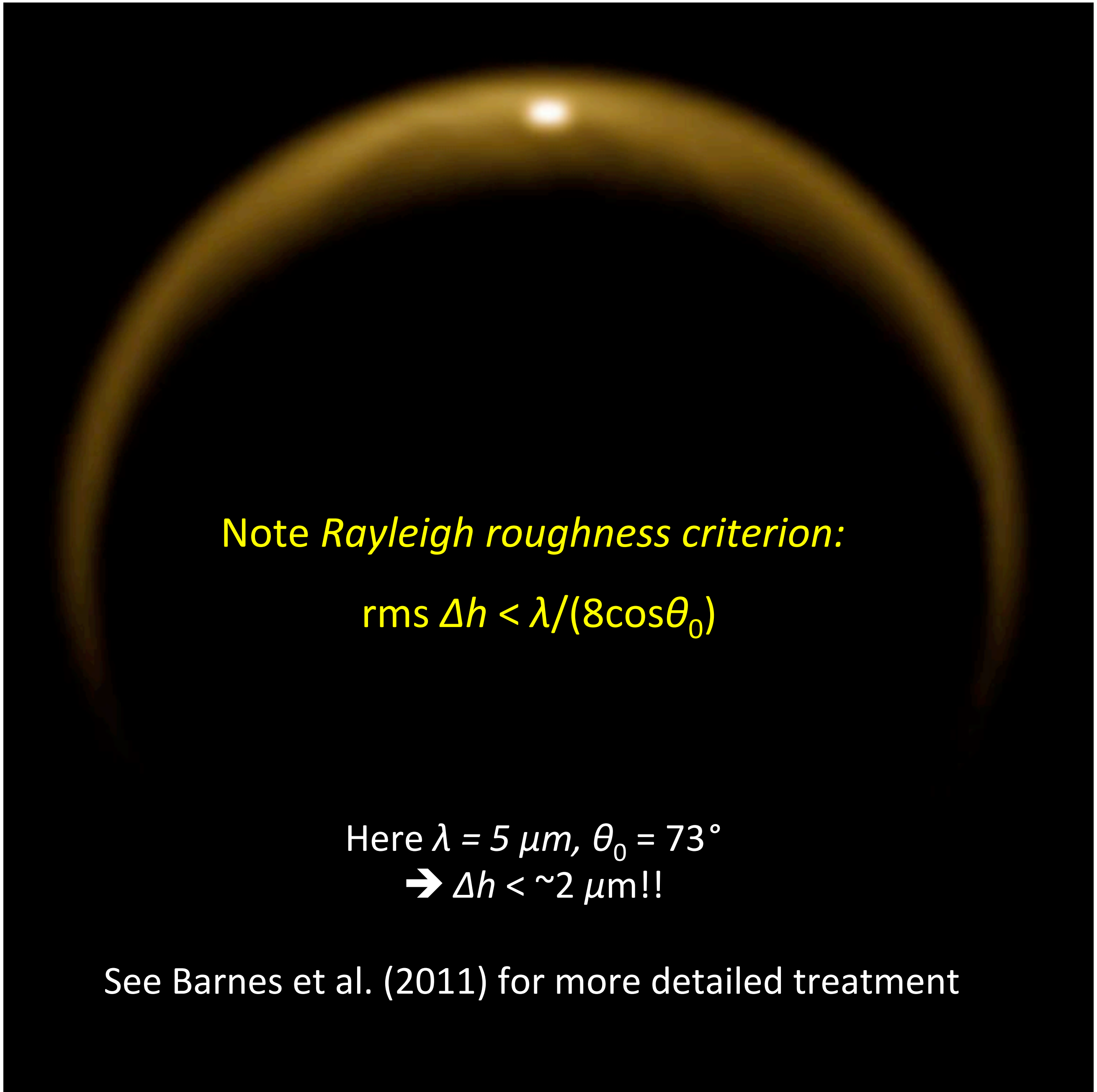
Henyey-Greenstein

BRDF ~ $(1 - g^2) / (1 - 2g\cos\Theta + g^2)^{3/2}$

Rees Fig. 3.10

Specular reflection: Earth

Specular reflection: Titan



Note *Rayleigh roughness criterion:*

$$\text{rms } \Delta h < \lambda/(8\cos\theta_0)$$

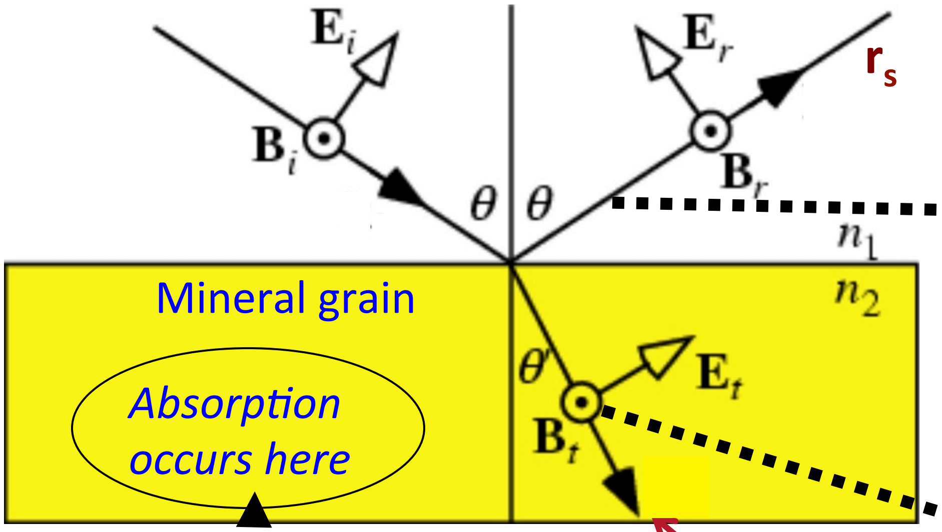
Here $\lambda = 5\ \mu m$, $\theta_0 = 73°$

➔ $\Delta h <$ ~2 μm!!

See Barnes et al. (2011) for more detailed treatment

*Light is **r**eflected, **a**bsorbed , or **t**ransmitted*

The amount of specular (mirror) reflection is given by Fresnel's Law



Transmitted component

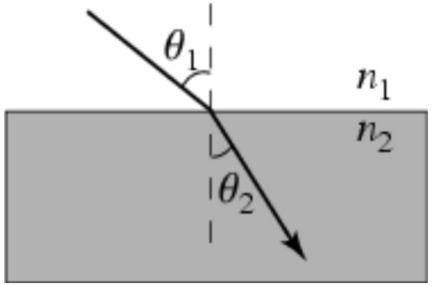
Fresnel's law

$$r_s = \frac{(n-1)^2 + K^2}{(n+1)^2 + K^2}$$

n = refractive index
K = extinction coefficient for the solid
r_s = fraction of light reflected from the 1st surface

Beer's law:
$(L = L_o e^{-kz})$

z = thickness of absorbing material
k = absorption coefficient for the solid
L_o = incoming directional radiance
L = outgoing radiance

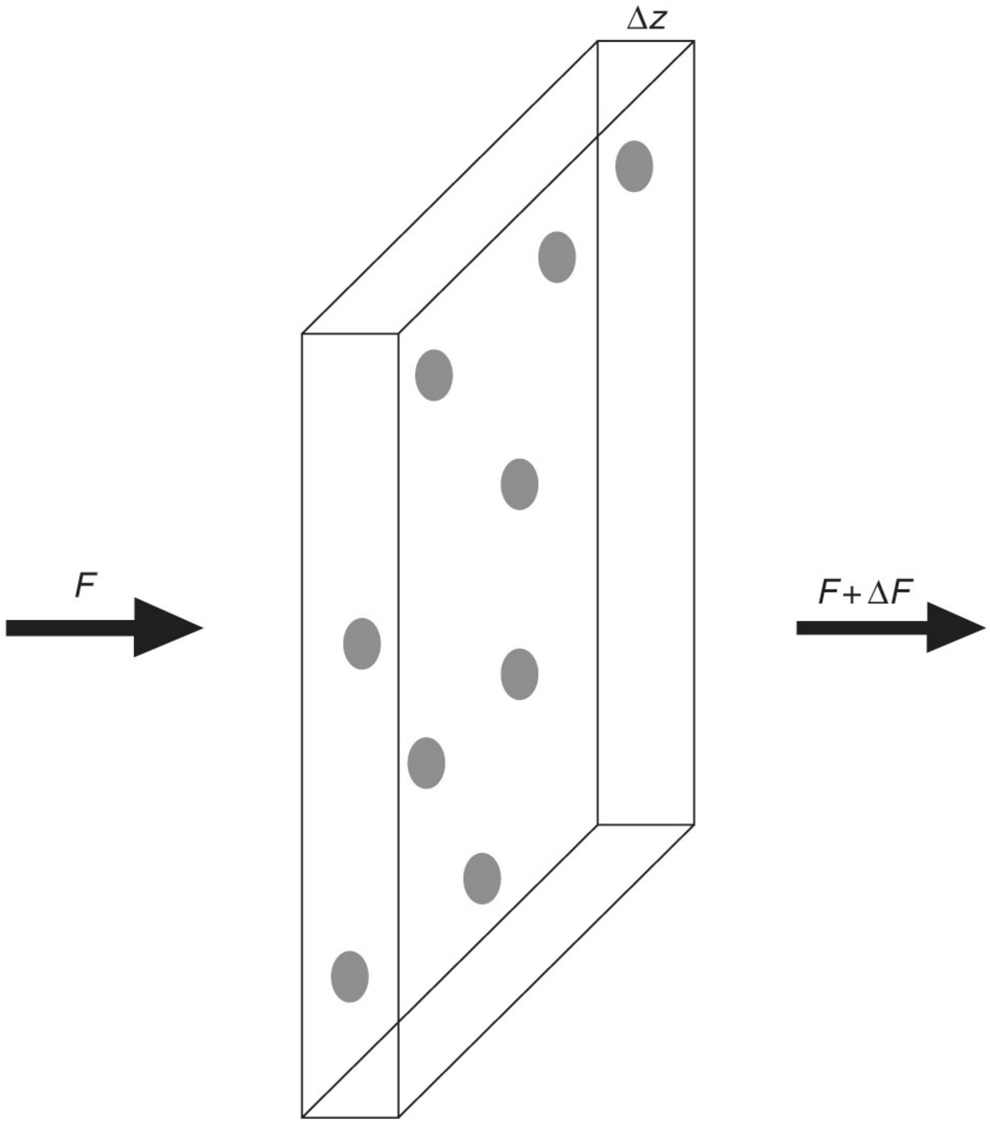


Snell's law:
$n_1 \cdot \sin\theta_1 = n_2 \cdot \sin\theta_2$

Light passing from one medium to another is *refracted* according to Snell's Law

$n = c/v$

Beer's law



Rees Fig. 3.27

$$\Delta F = -n\sigma_a F \Delta z$$

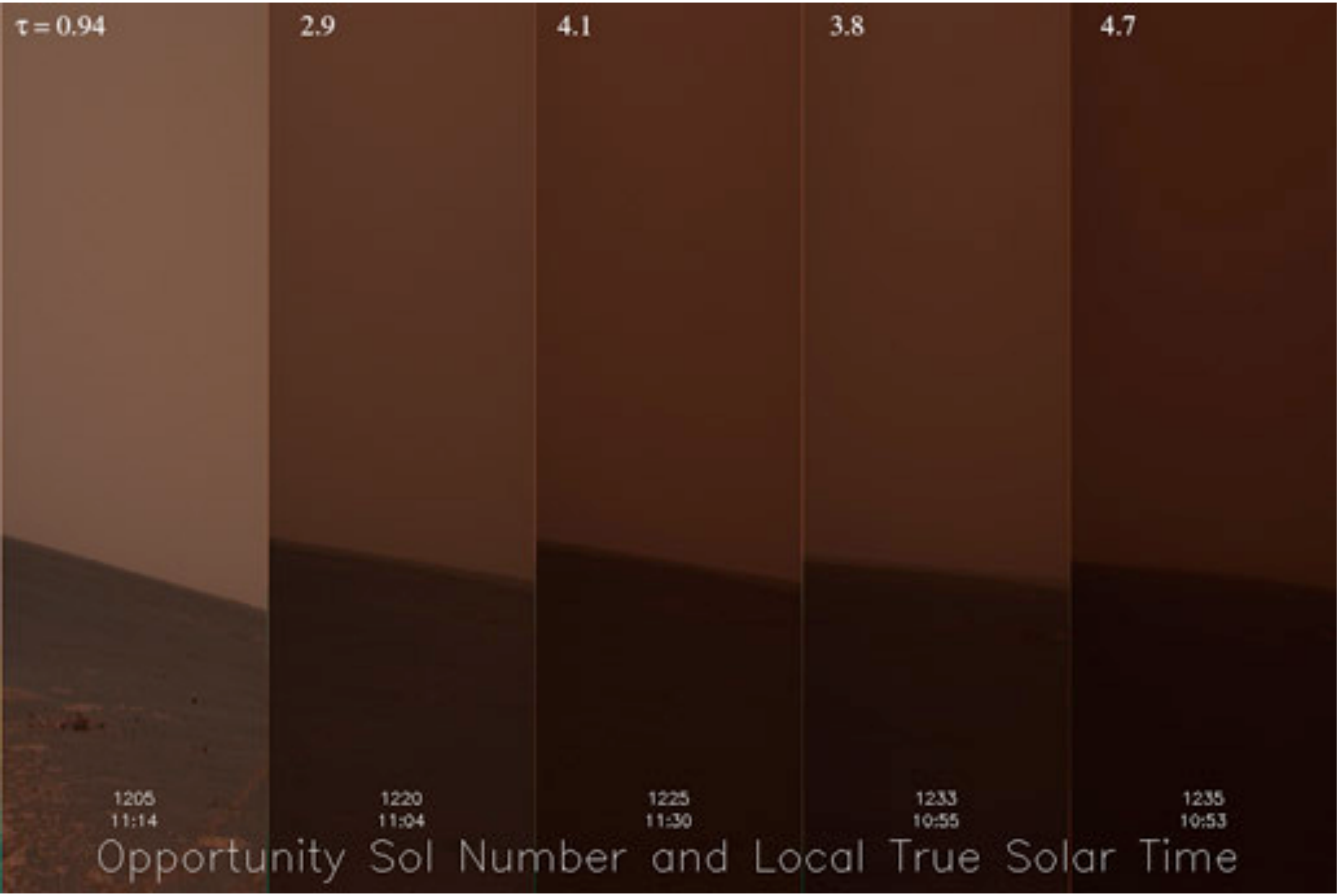
n = absorber # density
σ_a = absorption cross section

$n\sigma_a$ is the "absorption coefficient"
(1 / absorption length)

➔ $\boldsymbol{F = F_0 e^{-n\sigma z}}$

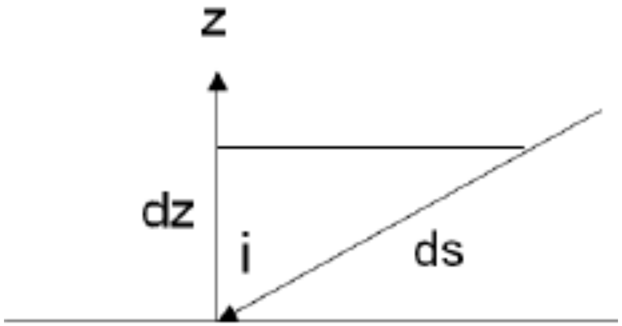
or $F = F_0 e^{-\tau}$
for "optical thickness" τ

Optical thickness: the 2007 Mars dust storm



Note regarding non-normal incidence

→ Longer path through the absorbing medium



$dz = ds \cos(i) = ds\ \mu$

$$F = F_0 e^{-n\sigma s} = F_0 e^{-n\sigma z/\mu}$$

$$i.e.,\ F = F_0 e^{-\tau/\mu}$$

Add emission

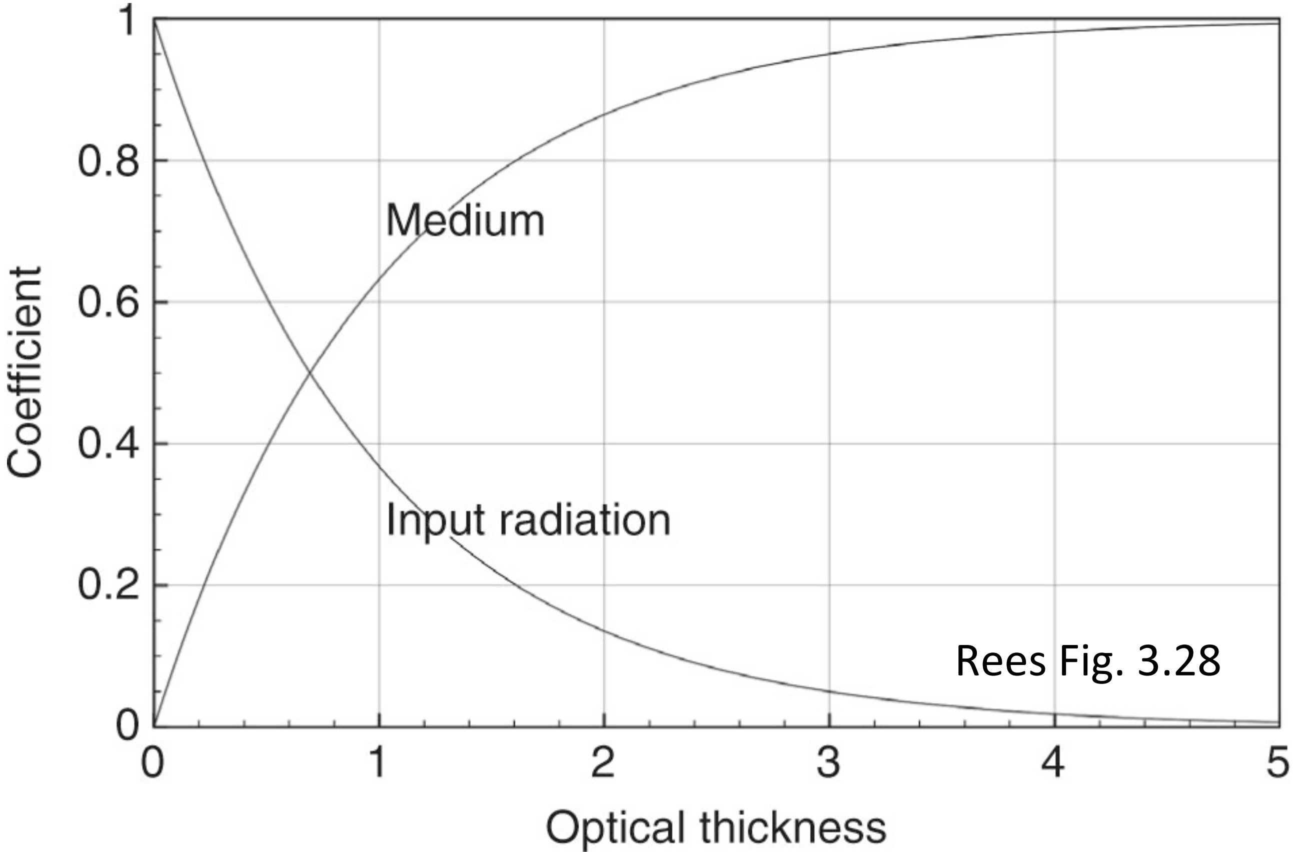
$$dL/dz = \gamma_a(B - L)$$

(why same coefficient on B and L?)

At low frequencies where B ~ T:

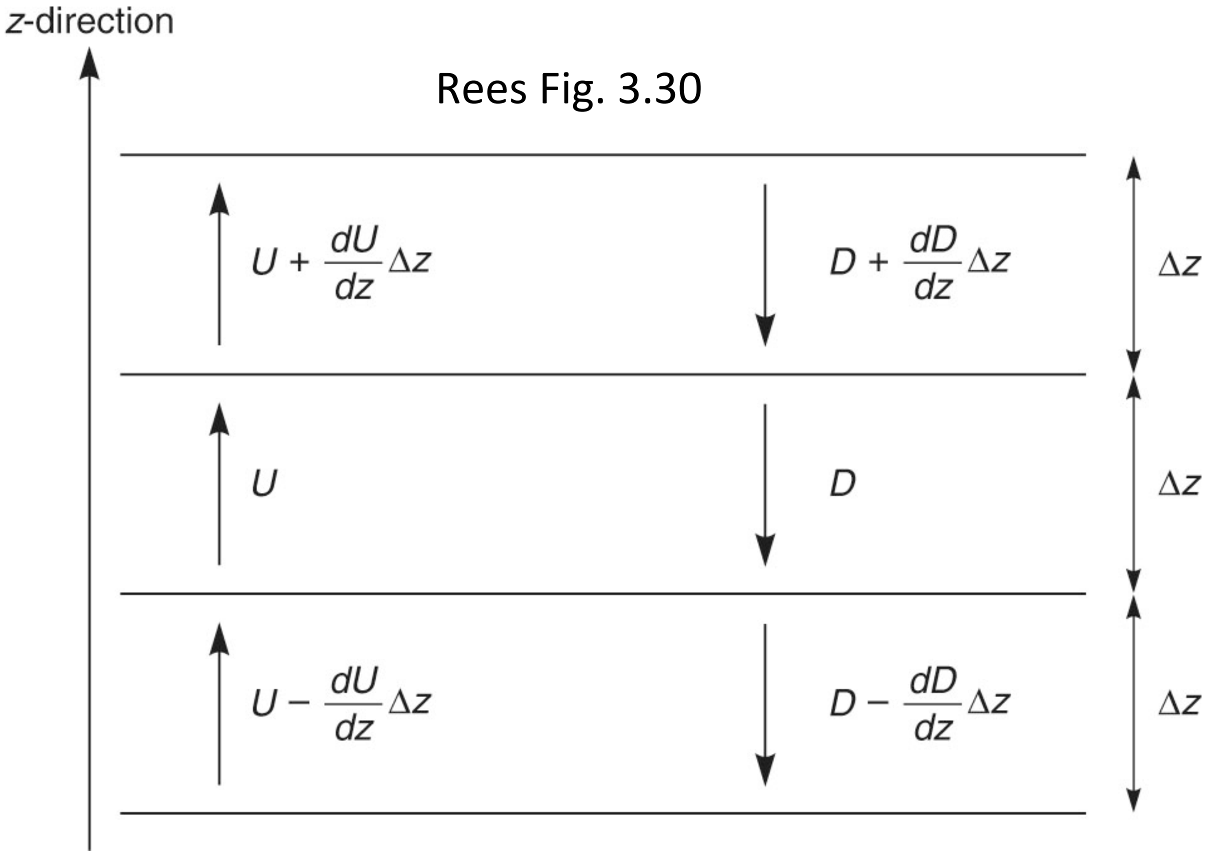
$$T_b = T_b(0)e^{-\tau} + T(1 - e^{-\tau})$$

observed input medium



Rees Fig. 3.28

Add scattering (reflection, in 2-stream case)


Rees Fig. 3.30

$$dU/dz = -(\gamma_a + \gamma_s)U + \gamma_s D$$

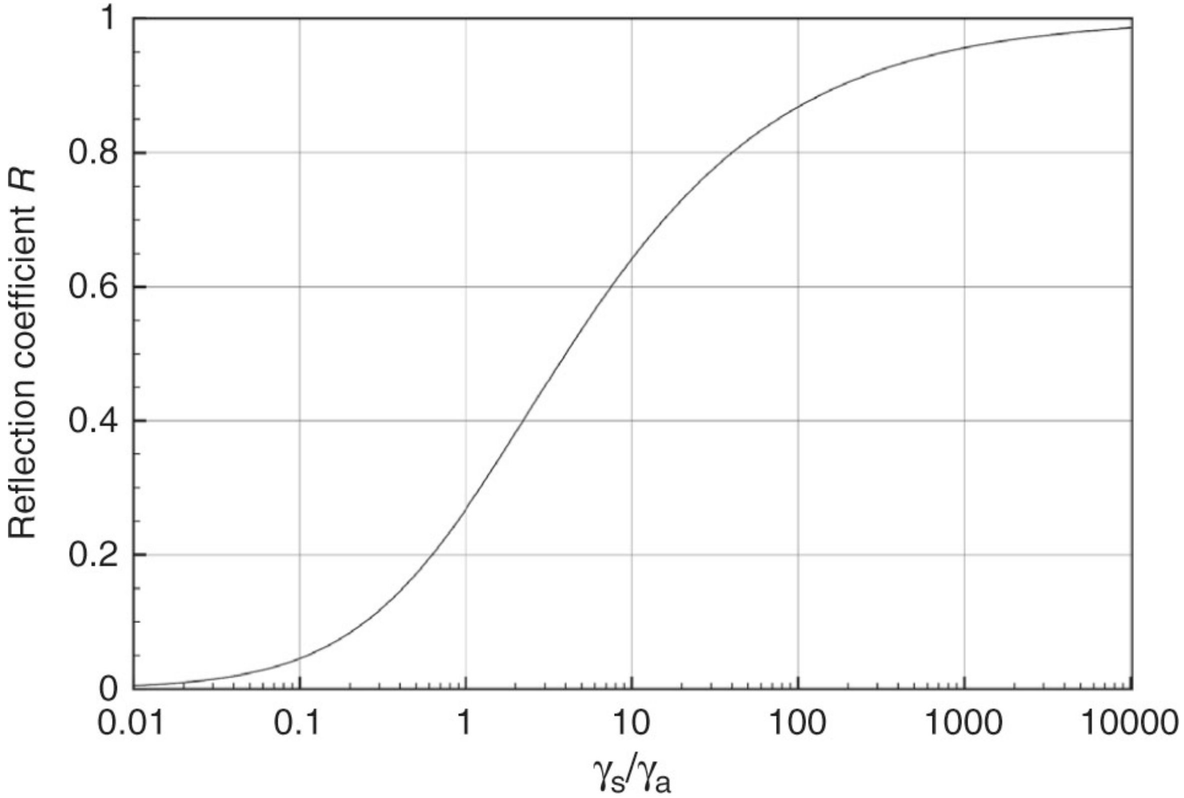
$$dD/dz = (\gamma_a + \gamma_s)D - \gamma_s U$$

Define ***extinction coefficient*** $\gamma_e = \gamma_a + \gamma_s$

Single scattering albedo *is* γ_s/γ_e

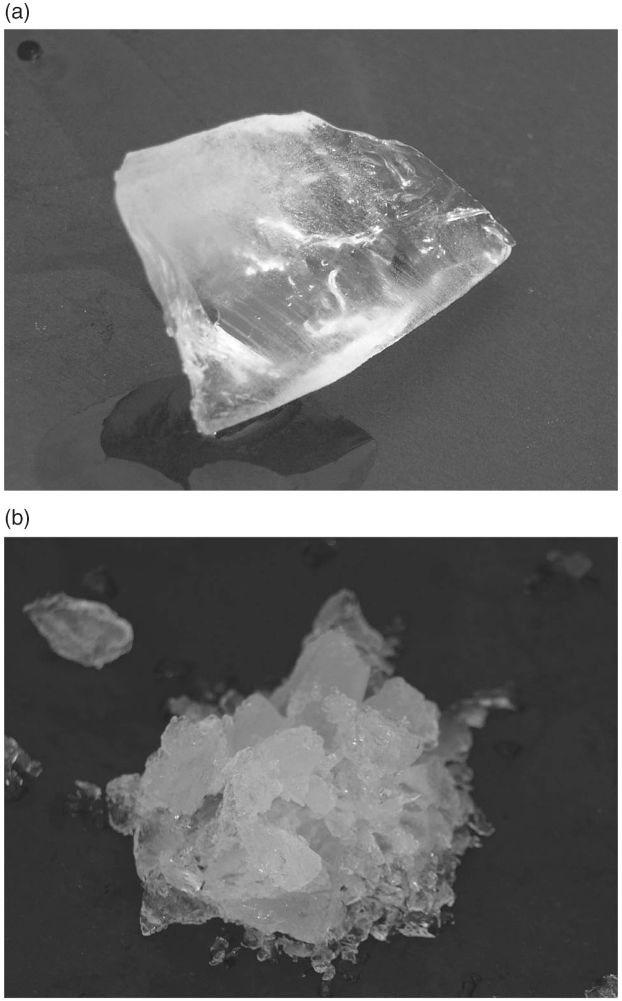
Scattering vs. absorption → reflectance

$$R = \gamma_a/\gamma_s + 1 - \sqrt{\gamma_a^2/\gamma_s^2 + 2\gamma_a/\gamma_s}$$



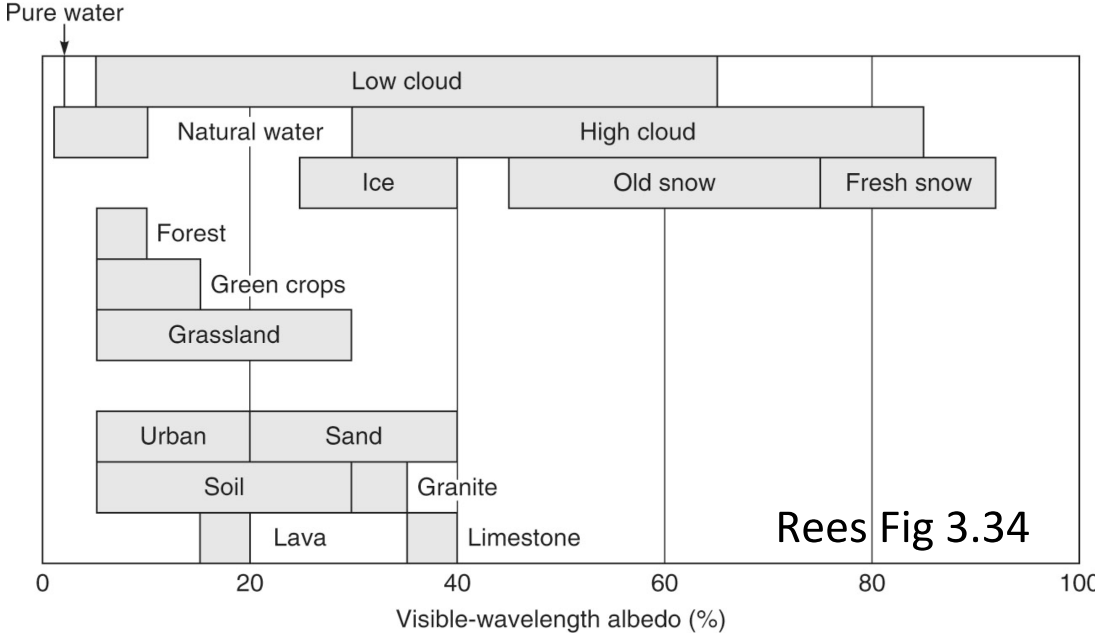
Rees Fig 3.31

Scattering vs. absorption → grain size matters



Rees Fig 3.32

Highly transmissive materials become highly reflective when you increase the density of scattering interfaces (ice, salt, clouds)



Rees Fig 3.34

Putting it all together

$dU/dz = -(\gamma_a + \gamma_s)U + \gamma_s D$

$dL/dz = \gamma_a(B - L)$

$$\frac{dLf(\theta,\phi)}{dz} = -(\gamma_a + \gamma_s)L_f(\theta,\phi) + \frac{\gamma_s}{4\pi}\int_{4\pi} L_f(\theta',\phi')p(\cos\Theta)\,d\Omega' + \gamma_a B_f$$

$p(\cos\Theta)$ is the scattering ***phase function; e.g.:***

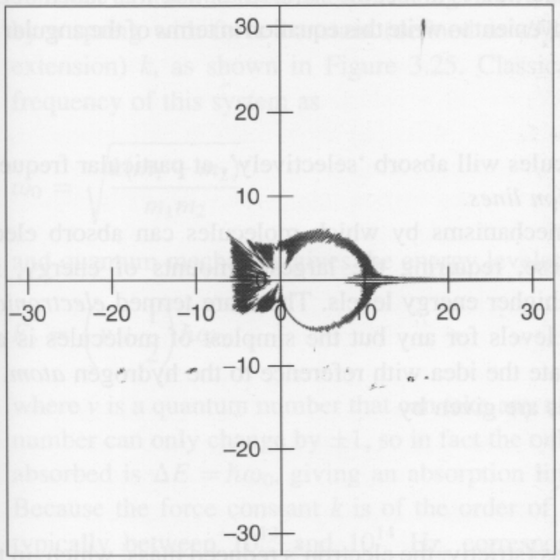


Figure 3.23. Phase function for light scattered from a spherical water droplet with a radius of 0.1 mm. The upper and lower halves of the diagram represent different polarisation states. The phase function is plotted on a logarithmic scale, and is somewhat schematic since not all details can be resolved. Note the strong forward scattering and the peak at about 140° corresponding to the primary rainbow.